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ON SOME SUBGROUPS ASSOCIATED WITH THE TENSOR SQUARE OF A GROUP

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ABSTRACT. In this paper we present some results about subgroup which is generalization of the subgroup $R_2^\otimes(G) = \{a \in G \mid [a, g] \otimes g = 1_\otimes, \forall g \in G\}$ of right 2_\otimes -Engel elements of a given group G . If p is an odd prime, then with the help of these results, we obtain some results about tensor squares of p -groups satisfying the law $[x, g, y] \otimes g = 1_\otimes$, for all $x, g, y \in G$. In particular p -groups satisfying the law $[x, g, y] \otimes g = 1_\otimes$ have abelian tensor squares. Moreover, we can determine tensor squares of two-generator p -groups of class three satisfying the law $[x, g, y] \otimes g = 1_\otimes$.

1. Introduction

Throughout this paper p is an odd prime and a p -group is a group in which every element has order a power of p .

For any group G , the non-abelian tensor square $G \otimes G$ is a group generated by the symbols $g \otimes h$, subject to the relations,

$$\begin{aligned}
 gg' \otimes h &= (g^{g'} \otimes h^{g'})(g' \otimes h), \\
 g \otimes hh' &= (g \otimes h')(g^{h'} \otimes h^{h'}),
 \end{aligned}$$

for all $g, g', h, h' \in G$, where $g^h = h^{-1}gh$ is conjugation on the right. The non-abelian tensor square is a special case of the non-abelian tensor product which has its origins in homotopy theory. It was introduced by R. Brown and J. L. Loday in [3] and [4], extending ideas of J. H. C. Whitehead in [13]. In [2], R. Brown, D. L. Johnson, and E. F. Robertson start the investigation of non-abelian tensor squares as group theoretical objects.

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The set of all elements a in G such that $a \otimes g = 1_{\otimes}$, for all g in G , is called *the tensor centre* of G and denoted by $Z^{\otimes}(G)$. This concept was introduced by G. J. Ellis [5].

Following [9] *The set of right n_{\otimes} -Engel elements* of a group G is defined as

$$R_n^{\otimes}(G) = \{a \in G : [a,_{n-1}x] \otimes x = 1_{\otimes} \quad \forall x \in G\}.$$

Here commutators are denoted by $[x, y] = x^{-1}y^{-1}xy$ and $[x_1, \dots, x_n, x_{n+1}] = [[x_1, \dots, x_n], x_{n+1}]$. Also for $x \in G$, the set of all x^g for $g \in G$ is called, *the conjugacy class* of x in G and is denoted by x^G .

The set of right n_{\otimes} -Engel elements in groups have been studied by several authors (see for example [1], [9], [10]). One of the results of [1] shows that $R_2^{\otimes}(G)$ is always a characteristic subgroup of G . Moravec in [9] investigated the properties of right 2_{\otimes} -Engel elements of a group G . The subgroup $B^{\otimes}(G)$ studied in this paper is a generalization of $R_2^{\otimes}(G)$.

Definition 1.1. *Let G be a group. We define*

$$C^{\otimes}(G) = \{a \in G | [ay, g, x] \otimes g = [y, g, x] \otimes g \quad \forall g, y, x \in G\},$$

$$B^{\otimes}(G) = \{a \in G | [a, g, x] \otimes g = 1_{\otimes} \quad \forall g, x \in G\}.$$

One can easily check that $C^{\otimes}(G)$ is a characteristic subgroup of G . It will be shown that $B^{\otimes}(G) = C^{\otimes}(G)$. Thus $B^{\otimes}(G)$ is a characteristic subgroup of G .

At first, we determine some information about $B^{\otimes}(G)$. Next we show that every p-group satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$ has an abelian tensor square and $\gamma_3(G) \leq Z^{\otimes}(G)$. With the help of this we can compute tensor squares of two-generator p-groups of class 3 satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$.

2. PRELIMINARY RESULTS

In this section we summarize some basic results which will be used in the proof of our main results.

Lemma 2.1. ([12, 5.1.5]) *Let G be a group and $x, y, z \in G$. Then*

$$[xy, z] = [x, z]^y[y, z] = [x, z][x, z, y][y, z]. \tag{2.1.1}$$

$$[x, yz] = [x, z][x, y]^z = [x, z][x, y][x, y, z]. \tag{2.1.2}$$

$$[g^{-1}, h]^g = [g, h]^{-1} = [g, h^{-1}]^h. \tag{2.1.3}$$

$$[x, y^{-1}, z]^y[y, z^{-1}, x]^z[z, x^{-1}, y]^x = 1. \tag{2.1.4}$$

Lemma 2.2. ([2]) *Let $g, g', h, h' \in G$. The following relations hold in $G \otimes G$:*

$$(g^{-1} \otimes h)^g = (g \otimes h)^{-1} = (g \otimes h^{-1})^h. \tag{2.2.1}$$

$$(g' \otimes h')^{g \otimes h} = (g' \otimes h')^{[g, h]}. \tag{2.2.2}$$

$$[g, h] \otimes g' = (g \otimes h)^{-1}(g \otimes h)^{g'}. \tag{2.2.3}$$

$$g' \otimes [g, h] = (g \otimes h)^{-g'}(g \otimes h). \tag{2.2.4}$$

$$[g, h] \otimes [g', h'] = [g \otimes h, g' \otimes h']. \tag{2.2.5}$$

Proposition 2.3. ([2]) *For a given group G , there exists a homomorphism*

$\lambda : G \otimes G \rightarrow G'$ *such that $\lambda(g \otimes h) = [g, h]$, for all $g, h \in G$.*

Lemma 2.4. ([1,9]) Let G be a group, $a \in R_2^\otimes(G)$ and $x, y, z \in G$. Then

$$[a, x] \otimes y = ([a, y] \otimes x)^{-1}. \quad (2.4.1)$$

$$[a, x]^z \otimes x = 1_\otimes. \quad (2.4.2)$$

$$[a, x] \in R_2^\otimes(G). \quad (2.4.3)$$

$$[a, x] \otimes [y, z] = 1_\otimes. \quad (2.4.4)$$

$$[x, y] \otimes a = ([x, a] \otimes y)^2 \text{ and } a \otimes [x, y] = ([a, x] \otimes y)^2. \quad (2.4.5)$$

$$([a, x, y] \otimes z)^2 = 1_\otimes. \quad (2.4.6)$$

Theorem 2.5. ([7]) The variety of groups which are nilpotent of class n or less (where n is a fixed integer greater than 2) may be defined by the law

$$[x, y_1, \dots, y_{n-1}, x] = 1.$$

3. RESULTS FOR $B^\otimes(G)$

The goal of this section is to prove the following results for $B^\otimes(G)$, which are tensor analogues to the work of Kappe in [6].

Lemma 3.1. Let G be a group, $g, x, d, d', d'' \in G$, $i, j \in \mathbb{Z}$ and $a \in B^\otimes(G)$. Then

$$[a, g, x]^d \otimes g^{d'} = 1_\otimes. \quad (3.1.1)$$

$$[a, g^i, x]^d \otimes g^{d'} = 1_\otimes. \quad (3.1.2)$$

$$[[a, g^i]^d, x]^{d'} \otimes (g^j)^{d''} = 1_\otimes. \quad (3.1.3)$$

$$[a, g, x]^d \otimes a^{d'} = 1_\otimes. \quad (3.1.4)$$

Proof. Commutator expansion gives,

$$\begin{aligned} 1_\otimes &= [a, g, xd] \otimes g \\ &= [a, g, d][a, g, x]^d \otimes g \\ &= ([a, g, d] \otimes g)^{[a, g, d]^d} ([a, g, x]^d \otimes g) \\ &= [a, g, x]^d \otimes g, \end{aligned}$$

lead to (3.1.1).

To prove (3.1.2) for $i > 0$, we use induction on i . (3.1.1) is as the basis of an inductive proof.

Now assume that (3.1.2) follows when i is a fixed positive k , we prove that (3.1.2) is true for $i = k + 1$.

$$\begin{aligned} [a, g^{k+1}, x]^d \otimes g^{d'} &= [a, g^k.g, x]^d \otimes g^{d'} \\ &= [[a, g][a, g^k]^g, x]^d \otimes g^{d'} \\ &= ([a, g, x]^w [[a, g^k]^g, x])^d \otimes g^{d'} \\ &= ([a, g, x]^{wd} \otimes g^{d'})^{w'} ([a, g^k]^g, x)^d \otimes g^{d'} \\ &= [a, g^k, x^{g^{-1}}]^{gd} \otimes g^{d'} \\ &= 1_\otimes, \end{aligned}$$

where $w = [a, g^k]^g$ and $w' = [[a, g^k]^g, x]^d$.

If $i < 0$, then put $t = -i$. Hence,

$$\begin{aligned}
 [a, g^i, x]^d \otimes g^{d'} &= [a, g^{-t}, x]^d \otimes g^{d'} \\
 &= [[a, g^t]^{-g^{-t}}, x]^d \otimes g^{d'} \quad \text{by(2.1.3)} \\
 &= [[a, g^t]^{-1}, x^{g^t}]^{g^{-t}d} \otimes g^{d'} \\
 &= [a, g^t, x^{g^t}]^{-wd} \otimes g^{d'} \quad \text{by(2.1.3)} \\
 &= ([a, g^t, x^{g^t}]^{wd} \otimes g^{d'})^{-w'} \quad \text{by(2.2.1)} \\
 &= 1_{\otimes},
 \end{aligned}$$

where $w = [a, g^t]^{-1}g^{-t}$ and $w' = [a, g^t, x^{g^t}]^{-wd}$.

These imply (3.1.2).

(3.1.3) is proved in a way similar to (3.1.2).

For the proof of (3.1.4) note that,

$$\begin{aligned}
 1_{\otimes} &= [a, ag, x]^d \otimes (ag)^{d'} \\
 &= [[a, ag], x]^d \otimes a^{d'} g^{d'} \\
 &= ([a, g, x]^d \otimes g^{d'})([a, g, x]^d \otimes a^{d'})^{g^{d'}} \\
 &= ([a, g, x]^d \otimes a^{d'})^{g^{d'}}, \quad \text{by(3.1.1)}
 \end{aligned}$$

proving (3.1.4). □

Theorem 3.2. *Let G be a group. Then we have*

$$B^{\otimes}(G) = C^{\otimes}(G). \quad (3.2.1)$$

$$R_2^{\otimes}(G) \subseteq B^{\otimes}(G). \quad (3.2.2)$$

$$[a, g, x, h] \otimes g = 1_{\otimes}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, h \in G. \quad (3.2.3)$$

$$[a, g, x, h] \otimes h = 1_{\otimes}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, h \in G. \quad (3.2.4)$$

$$([a, g, x, b, c] \otimes h)^2 = 1_{\otimes}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, b, c, h \in G. \quad (3.2.5)$$

$$[a, g, x] \otimes h = ([a, h, x] \otimes g)^{-1}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, h \in G. \quad (3.2.6)$$

Proof. To prove (3.2.1) let $a \in B^{\otimes}(G)$. Then $[[a, g]^y, x]^{[y, g]} \otimes g = 1_{\otimes}$ by (3.1.3). Hence,

$$\begin{aligned}
 [ay, g, x] \otimes g &= ([[a, g]^y, x]^{[y, g]} \otimes g)^{[y, g, x]}([y, g, x] \otimes g) \\
 &= [y, g, x] \otimes g
 \end{aligned}$$

i.e., $B^{\otimes}(G) \subseteq C^{\otimes}(G)$ and hence (3.2.1) follows since clearly $C^{\otimes}(G) \subseteq B^{\otimes}(G)$.

To prove (3.2.2), let $a \in R_2^{\otimes}(G)$. Then by proposition 2.3, $[a, g, g] = 1$. Hence, by (2.4.3) and (2.4.1)

we have

$$\begin{aligned} [a, g, x] \otimes g &= ([[a, g], g] \otimes x)^{-1} \\ &= (1 \otimes x)^{-1} \\ &= 1_{\otimes}, \end{aligned}$$

proving $R_2^{\otimes}(G) \subseteq B^{\otimes}(G)$.

To prove (3.2.3), we have

$$\begin{aligned} [a, g, x, h] \otimes g &= [[a, g]^{-1}[a, g]^x, h] \otimes g \\ &= [a, g, h]^{-w} [[a, g]^x, h] \otimes g \\ &= ([a, g, h]^w \otimes g)^{-w'} ([[a, g]^x, h] \otimes g) \\ &= 1_{\otimes}, \quad \text{by(3.1.3)} \end{aligned}$$

where $w = [a, g]^{-1}[a, g]^x$ and $w' = [a, g, h]^{-w} [[a, g]^x, h]$.

To prove (3.2.4), since $a \in B^{\otimes}(G)$, then by proposition 2.3 we have $1 = [a, gh, x, gh]$. Hence, commutator expansion gives

$$\begin{aligned} 1 &= [[a, h][a, g]^h, x, gh] \\ &= [[a, h, x]^{[a, g]^h} [a, g]^h, x], gh] \\ &= [y_1 y_2, gh], \end{aligned}$$

where $y_1 = [a, h, x]^{[a, g]^h}$ and $y_2 = [[a, g]^h, x]$. Now by (3.1.3), $y_1 \otimes h = 1_{\otimes}$ and $y_2 \otimes g = 1_{\otimes}$. Hence, by proposition 2.3 we have $[y_1, h] = 1$ and $[y_2, g] = 1$. Then the commutator expansion of $1 = [y_1 y_2, gh]$ gives

$$1 = [y_1, gh]^{y_2} [y_2, gh] = [y_1, h]^{y_2} [y_1, g]^{hy_2} [y_2, h] [y_2, g]^h.$$

Hence, $1 = [y_1, g]^{hy_2} [y_2, h]$ and so

$$\begin{aligned} 1_{\otimes} &= [y_1, g]^{hy_2} [y_2, h] \otimes h \\ &= ([y_1, g]^{hy_2} \otimes h)^{[y_2, h]} ([y_2, h] \otimes h). \end{aligned}$$

Now we prove $[y_1, g]^{hy_2} \otimes h = 1_{\otimes}$. We have

$$\begin{aligned} 1_{\otimes} &= y_1^{ghy_2} \otimes h \quad \text{by(3.1.3)} \\ &= (y_1 [y_1, g])^{hy_2} \otimes h \\ &= (y_1^{hy_2} \otimes h)^{[y_1, g]^{hy_2}} ([y_1, g]^{hy_2} \otimes h) \\ &= [y_1, g]^{hy_2} \otimes h. \quad \text{by(3.1.3)} \end{aligned}$$

Hence, we obtain

$$1_{\otimes} = [y_2, h] \otimes h = [[a, g]^h, x, h] \otimes h.$$

The substitution of x^h for x finally gives

$$1_{\otimes} = [[a, g]^h, x^h, h] \otimes h = ([a, g, x, h] \otimes h)^h,$$

proving (3.2.4).

To prove (3.2.5), $[a, g, x] \in R_2^{\otimes}(G)$ by (3.2.4). Hence, (3.2.5) follows by (2.4.6).

Finally for (3.2.6), expansion of $1_{\otimes} = [a, gh, x] \otimes gh$, as in the proof (3.2.4), leads to

$$1_{\otimes} = (y_1 \otimes g)^{hy_2}(y_2 \otimes h),$$

where $y_1 = [a, h, x]^{[a, g]^h}$ and $y_2 = [[a, g]^h, x]$. Now by (3.1.4) we have $[a, h, x] \otimes [a, g]^h = 1_{\otimes}$, since $[a, g]^h \in \langle a^G \rangle$. Hence, $[[a, h, x], [a, g]^h] = 1$ by proposition 2.3. Therefore $y_1 = [a, h, x]$. Now we have

$$\begin{aligned} (y_1 \otimes g)^{hy_2} &= ([a, h, x] \otimes g)([[a, h, x], g] \otimes hy_2) \quad \text{by (2.2.3)} \\ &= ([a, h, x] \otimes g)([[a, h, x], g] \otimes y_2)([[a, h, x], g] \otimes h)^{y_2} \\ &= ([a, h, x] \otimes g)([[a, h, x], g] \otimes y_2) \quad \text{by (3.2.3)} \\ &= [a, h, x] \otimes g \quad \text{by (3.1.4), since } y_2 \in \langle a^G \rangle. \end{aligned}$$

To simplify $y_2 \otimes h$, write $[a, g]^h = [a, g][a, g, h]$ and expand

$$\begin{aligned} y_2 \otimes h &= [a, g, x]^{w_0}[a, g, h, x] \otimes h \\ &= ([a, g, x]^{w_0} \otimes h)^{w_1}([a, g, h, x] \otimes h) \\ &= ([a, g, x]^{w_0} \otimes h)^{w_1} \quad \text{since } [a, g] \in B^{\otimes}(G) \\ &= ([a, g, x]^{w_0} \otimes h)([a, g, x]^{w_0}, h] \otimes w_1), \quad \text{by (2.2.3)} \end{aligned}$$

where $w_0 = [a, g, h]$ and $w_1 = [w_0, x]$. Now $[a, g, x] \otimes w_0 = 1_{\otimes}$ by (3.1.4), since $w_0 \in \langle a^G \rangle$. Hence, $[[a, g, x], w_0] = 1$ by proposition 2.3. Therefore $[a, g, x]^{w_0} = [a, g, x]$. Also $([a, g, x]^{w_0}, h] \otimes w_1) = 1_{\otimes}$ by (3.1.4), since $w_1 \in \langle a^G \rangle$. Hence,

$$y_2 \otimes h = [a, g, x] \otimes h.$$

Now, altogether we have

$$1_{\otimes} = (y_1 \otimes g)^{hy_2}(y_2 \otimes h) = ([a, h, x] \otimes g)([a, g, x] \otimes h),$$

proving (3.2.6). □

4. The variety with the tensor square

A variety is a class of groups defined by laws, or identical relations.

Lemma 4.1. *Every group G of the variety with the law*

$$[x, g, y] \otimes g = 1_{\otimes}, \quad (4.1.1)$$

for all $x, g, y \in G$ is nilpotent of class at most 3.

Proof. Expansion of $1_{\otimes} = [x, xg, y] \otimes xg$ implies that

$$1_{\otimes} = [x, g, y] \otimes x.$$

Thus G is nilpotent of class at most 3 by proposition 2.3 and theorem 2.5. □

Proposition 4.2. *Let G be a group satisfying the law (4.1.1) and $a, b, x, y \in G$. Then*
 $G' \subseteq R_2^{\otimes}(G).$ (4.2.1)

$$[a, b] \otimes [x, y] = ([a, b, x] \otimes y)^2 \text{ and } [a, b] \otimes [x, y] = ([a, [x, y]] \otimes b)^2. \quad (4.2.2)$$

$$([a, b] \otimes [x, y])^2 = 1_{\otimes}. \quad (4.2.3)$$

Proof. To prove (4.2.1), by (2.1.4) and lemma 4.1 we have

$$[x^{-1}, y^{-1}, z][y, z^{-1}, x^{-1}][z, x, y] = 1,$$

for all $x, y, z \in G$. Hence, $1_{\otimes} = [x^{-1}, y^{-1}, z][y, z^{-1}, x^{-1}][z, x, y] \otimes y^{-1}$. The equation yields

$$1_{\otimes} = [z, x, y] \otimes y,$$

proving (4.2.1).

Next, (4.2.2) follows from (4.2.1) and (2.4.5).

Finally for (4.2.3), we have

$$\begin{aligned} [a, b] \otimes [x, y] &= ([x, y, a]^{-1} \otimes b)^2 && \text{by (4.2.2)} \\ &= ([x, y, a] \otimes b)^{-2} && \text{by (2.2.1), (2.2.3) and 4.1} \\ &= ([x, y, b] \otimes a)^2 && \text{by (4.2.1) and (2.4.1)} \\ &= ([x, a, b] \otimes y)^{-2} && \text{by (3.2.6)} \\ &= ([a, x, b] \otimes y)^2 && \text{by (2.2.1), (2.2.3) and 4.1} \\ &= ([a, x, y] \otimes b)^{-2} && \text{by (4.2.1) and (2.4.1)} \\ &= ([a, b, y] \otimes x)^2 && \text{by (3.2.6)} \\ &= ([a, b, x] \otimes y)^{-2} && \text{by (4.2.1) and (2.4.1)} \\ &= ([a, b] \otimes [x, y])^{-1}, && \text{by (4.2.2)} \end{aligned}$$

proving, (4.2.3) □

Theorem 4.3. *A group G has a finite covering by subgroups satisfying the law (4.1.1) if and only if $|G : B^{\otimes}(G)| < \infty$.*

Proof. Suppose that $G = \bigcup_{i=1}^n H_i$ where H_i are subgroups of G satisfying the law (4.1.1). The result of B. H. Neumann [11], shows that we may assume that $|G : H_i| < \infty$ for every i . Hence, the subgroup $D = \bigcap_{i=1}^n H_i$ has a finite index in G . It is clear that $D \leq B^{\otimes}(G)$ and hence, $|G : B^{\otimes}(G)| < \infty$.

Conversely, let $\{g_1, \dots, g_n\}$ be a transversal of $B^{\otimes}(G)$ in G and let $H_i = \langle g_i \rangle B^{\otimes}(G)$. We have $G = \bigcup_{i=1}^n H_i$, hence it suffices to prove that each H_i satisfying the law (4.1.1).

Let $x = g^i a, y = g^j b$ and z be arbitrary elements of $\langle g \rangle B^{\otimes}(G)$, where $i, j \in \mathbb{Z}$ and $a, b \in B^{\otimes}(G)$. Since $a, b \in B^{\otimes}(G)$, using (3.1.4), we obtain,

$$\begin{aligned} [x, y, z] \otimes y &= [g^i a, g^j b, z] \otimes g^j b \\ &= [[g^i, g^j b]^a [a, g^j b], z] \otimes g^j b \end{aligned}$$

$$\begin{aligned}
 &= [[g^i, g^j b]^a, z]^w [a, g^j b, z] \otimes g^j b \\
 &= ([[g^i, g^j b]^a, z]^w \otimes g^j b)^{w'} ([a, g^j b, z] \otimes g^j b) \\
 &= ([[g^i, b]^a, z]^w \otimes b)^{w'} ([[g^i, b]^a, z]^w \otimes g^j)^{bw'} \quad a \in B^\otimes(G) \\
 &= ([[g^i, b]^a, z]^w \otimes g^j)^{bw'} \quad b \in B^\otimes(G) \\
 &= ([[b, g^i]^a, z]^v \otimes g^j)^{-v'} \\
 &= 1_\otimes, \quad b \in B^\otimes(G)
 \end{aligned}$$

where $w = [a, g^j b]$, $w' = [a, g^j b, z]$, $v = [b, g^i]^{-a} w$, $v' = [[b, g^i]^a, z]^{-v} b w'$, as required. □

Theorem 4.4. *Let G be a p -group satisfying the law (4.1.1). Then*

$$G \otimes G \text{ is abelian.} \quad (4.4.1)$$

$$\gamma_3(G) \leq Z^\otimes(G). \quad (4.4.2)$$

Proof. If G is an abelian p -group, then the result is obvious. Otherwise, $G \otimes G$ has no element of order 2, since G is a p -group and p is an odd prime. Hence, $[a, b] \otimes [x, y] = 1_\otimes$ by (4.2.3). So (2.2.5) implies (4.4.1). Next we have $([a, b, x] \otimes y)^2 = 1_\otimes$ by (4.2.2) since $[a, b] \otimes [x, y] = 1_\otimes$. Thus $1_\otimes = [a, b, x] \otimes y$, proving (4.4.2). □

Corollary 4.5. *Let G be a two-generator p -group of class 3 satisfying the law (4.1.1). Then we can determine tensor square of G .*

Proof. If G is a p -group satisfying the law (4.1.1), then by (4.4.2), $\gamma_3(G) \leq Z^\otimes(G)$. Now using the result of G. J. Ellis [5], we see that $G \otimes G \cong \frac{G}{\gamma_3(G)} \otimes \frac{G}{\gamma_3(G)}$, hence the calculations of tensor squares reduce to the calculations of tensor squares of class 2 groups. From [8], we obtain the complete classification of tensor square of two-generator p -groups of class 2. Hence by Theorem 36 from [8]

$$G \otimes G \cong \begin{cases} C_{p^\alpha} \times C_{p^\beta} \times C_{p^\beta} \times C_{p^\beta} \times C_{p^\rho} \times C_{p^\rho} & \text{if } \rho \leq \sigma \\ C_{p^\alpha} \times C_{p^\beta} \times C_{p^\beta} \times C_{p^{\beta+\tau}} \times C_{p^\sigma} \times C_{p^\sigma} & \text{if } \rho > \sigma \end{cases}$$

where $\tau = \min(\alpha - \beta, \rho - \sigma)$ and $\alpha, \beta, \gamma, \rho$ and σ are defined integers as in Theorem 1 from [8](see [8] for more information). □

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