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A NOTE ON FINITE C-TIDY GROUPS

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Abstract. Let G be a group and $x \in G$. The cyclicizer of x is defined to be the subset $Cyc(x) = \{y \in G \mid \langle x, y\rangle \text{ is cyclic}\}$. G is said to be a tidy group if $Cyc(x)$ is a subgroup for all $x \in G$. We call G to be a C-tidy group if $Cyc(x)$ is a cyclic subgroup for all $x \in G \setminus K(G)$, where $K(G)$ is the intersection of all the cyclicizers in G. In this note, we classify finite C-tidy groups with $K(G) = \{1\}$.

1. Introduction

Let G be a group and $x \in G$. The centralizer of x, denoted by $C_G(x)$, is defined by $C_G(x) = \{y \in G \mid yx = xy\}$. The cyclicizer of x, denoted by $Cyc_G(x)$, is defined to be the subset $Cyc_G(x) = \{y \in G \mid \langle x, y\rangle \text{ is cyclic}\}$. If the context is clear we will write $C(x)$ and $Cyc(x)$ in place of $C_G(x)$ and $Cyc_G(x)$ respectively. In general, $Cyc(x)$ is not a subgroup. For example, in the group $C_2 \times C_4$, we have

$$Cyc((0, 2)) = \{(0, 0), (0, 1), (0, 2), (0, 3), (1, 1), (1, 3)\},$$

which is not a subgroup of $C_2 \times C_4$. Following [15], a group G is said to be a tidy group if $Cyc(x)$ is a subgroup for all $x \in G$. We call G to be a C-tidy group if $Cyc(x)$ is a cyclic subgroup for all $x \in G \setminus K(G)$, where $K(G)$ is the intersection of all the cyclicizers in G.

Starting with D. Patrick and E. Wepsic in 1991, [16] many authors have studied and characterised groups (finite and infinite) in terms of cyclicizers and tidy properties (see [15], [2], [3], [6], [14]). In this paper, we continue with this problem and classify finite C-tidy groups G with $K(G) = \{1\}$.

Throughout this paper, all groups are finite and all notations are usual. For example, C_n denotes the cyclic group of order n and C_n^k denotes $C_n \times \cdots \times C_n$, k-times, where n and k are positive integers.

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$PGL(n, q)$ and $PSL(n, q)$ denote the projective general linear and the projective special linear group of degree n over the field of size q respectively, $Sz(2^{2m+1}), m > 0$ denotes the Suzuki group over the field with 2^{2m+1} elements. For a group G, 1 and $Z(G)$ denote the identity element and the center respectively.

2. Some basic results

We begin with some definitions concerning coverings and partitions of a group. It may be mentioned here that the first paper concerning partitions of groups was published in 1906 by G. A. Miller [13].

Definition: A collection Π of non-trivial subgroups of a group G is called a covering if every element of G belongs to a subgroup in Π. The covering Π is called a partition if every non-trivial element of G belongs to a unique subgroup in Π. If $|\Pi| = 1$, the partition is said to be trivial. The subgroups in Π are called the components of Π. A partition Π of a group G is said to be normal if $g^{-1}Xg \in \Pi$ for every $X \in \Pi$ and $g \in G$. Following Kontorovich, a group G is said to be completely decomposable if it has a partition Π such that every component of Π is cyclic. See ([19]) for more information in this regard.

Proposition 2.1. *Let G be a C-tidy group. Then for any $x, y \in G \setminus K(G)$ either $Cyc(x) = Cyc(y)$ or $Cyc(x) \cap Cyc(y) = K(G)$.*

Proof. Suppose $Cyc(x) \neq Cyc(y)$. Let $a \in Cyc(x) \cap Cyc(y) \setminus K(G)$. Then $a \in Cyc(x)$ and since $Cyc(x)$ is cyclic, therefore $Cyc(x) \subseteq Cyc(a)$. Similarly, $Cyc(a) \subseteq Cyc(x)$ and hence $Cyc(a) = Cyc(x)$. In the same way, we can show that $Cyc(a) = Cyc(y)$, which is a contradiction. □

We now prove the following proposition which will be used later.

Proposition 2.2. *Let G be a group with a partition Π. Then G is tidy (C-tidy) if and only if every component is tidy (C-tidy).*

Proof. Let $\Pi = \{X_i\}_{i\in I}$ and $x \in G \setminus \{1\}$. Then $x \in X_i$ for some $i \in I$. Suppose $y \in Cyc_G(x) \setminus \{1\}$. Then $\langle y, x\rangle = \langle t\rangle$ for some $t \in G$. Hence $x = t^k$ for some positive integer k. Suppose $t \notin X_i$. Then $t \in X_j$ for some $j \neq i$. Therefore $t^k \in X_i \cap X_j$, which is a contradiction. Hence $y \in X_i$ and so $Cyc_G(x) \subseteq X_i$. Now, since $Cyc_{X_i}(x) = Cyc_G(x) \cap X_i = Cyc_G(x)$ for any $x \in X_i \setminus \{1\}$ and $i \in I$, therefore the result follows. □

As a consequence of Proposition 2.1 and Proposition 2.2, we get the following equivalent condition for completely decomposable groups.

Proposition 2.3. *Let G be a non-cyclic group. Then G is C-tidy with $K(G) = \{1\}$ if and only if G is completely decomposable.*

Proof. Let G be C-tidy with $K(G) = \{1\}$. Let Π be the collection of all proper cyclicizers of G. By Proposition 2.1, Π is a partition of G such that every component of Π is cyclic. Hence G is completely decomposable.

Conversely, suppose G is completely decomposable. Then by Proposition 2.2, G is C-tidy with $K(G) = \{1\}$. □

A group G is said to be a Frobenius group if it contains a subgroup H such that $\{1\} \neq H \neq G$ and $H \cap H^g = \{1\}$ for all $g \in G \setminus H$. A subgroup with these properties is called a Frobenius complement of G. The Frobenius kernel of G, with respect to H, is defined by $K = (G \setminus \bigcup_{g\in G} H^g) \cup \{1\}$. It is well known that (see [17]) K is nilpotent and G has a partition $\{K, H^{g_1}, H^{g_2}, \ldots, H^{g_m}\}$ with $K = \{g_1, g_2, \ldots, g_m\}$, the so called Frobenius partition. By Problem 7.1 of [10], it follows that $H^{g_i}, 1 \leq i \leq m$ are Frobenius complements of G.

The following simple lemma will be used in proving the next proposition.

Lemma 2.4. *Let G be a Frobenius group with Frobenius complement H. Then H is a tidy group.*

Proof. By Lemma 3 , p. 273 of [5], every sylow subgroup of H is either cyclic or generalized quaternion. Therefore by Theorem 6 of [16], $Cyc_H(x) = C_H(x)$ for all $x \in H$. □

Proposition 2.5. *Let G be a Frobenius group with Frobenius kernel K. Then G is tidy if and only if K is tidy.*

Proof. If K is tidy, then by Proposition 2.4, every component of the Frobenius partition of G is tidy and by Proposition 2.2, G is tidy. Converse is trivial. □

In the following theorem, $I(G)$ denotes the set of all solutions of the equation $x^2 = 1$ in G.

Theorem 2.6 ((See [6], Theorem 2.4))**.** *The Suzuki group $Sz(2^{2m+1}), m > 0$ is not tidy.*

Proof. Let $G = Sz(2^{2m+1}), m > 0$ and S be a sylow 2-subgroup of G. By p. 12 of [7], $|Z(S)| = 2^{2m+1}, Z(S) = I(G)$ and S is of exponent 4. Therefore by Theorem 14 of [15], S is not tidy and hence G is not tidy. □

Let G be a group and p be a prime. We recall that the Hughes subgroup denoted by $H_p(G)$ is defined to be the subgroup generated by all the elements of G whose order is not p. The group G is said to be a group of Hughes-Thompson type if it is not a p-group and $H_p(G) \neq G$ for some prime p.

The following theorem gives a classification of all groups with a non-trivial partition (see [19]).

Theorem 2.7. *A group G has a non-trivial partition if and only if it satisfies one of the following conditions:*

(1) *G is a p-group with $H_p(G) \neq G$ and $|G| > p$, p being a prime,*
(2) *G is a Frobenius group,*

(3) *G is a group of Hughes-Thompson type,*
(4) *G is isomorphic with* $PGL(2, p^h)$, *p being an odd prime,* $h \geq 1$,
(5) *G is isomorphic with* $PSL(2, p^h)$, *p being a prime,* $h \geq 1$,
(6) *G is isomorphic with a Suzuki group* $Sz(2^{2m+1}), m > 0$.

3. C-tidy groups with $K(G) = \{1\}$

In this section we study the groups in which $Cyc(x)$ is a cyclic subgroup for all $x \in G \setminus \{1\}$. We begin with the following definition.

Definition: A group G is called a C-tidy group if $Cyc(x)$ is a cyclic subgroup for all $x \in G \setminus K(G)$.

Note that there exists tidy groups which are not C-tidy groups. For example, the alternating group A_7 is tidy (see Theorem 1.3 of [6]), but not C-tidy, because $(4\ 5)(6\ 7)$, $(4\ 6)(5\ 7) \in Cyc((1\ 2\ 3))$.

We now classify all nilpotent C-tidy groups with $K(G) = \{1\}$.

Theorem 3.1. *G is a nilpotent C-tidy group with* $K(G) = \{1\}$ *if and only if G is a non-cyclic p-group with a cyclic normal subgroup H such that* $o(x) = p$ *for all* $x \in G \setminus H$, *where p is a prime.*

Proof. Clearly G cannot be cyclic and we have $G = P \times A$, where P is a non-cyclic p-group and A is a group with $gcd(p, |A|) = 1$, p being a prime. By Lemma 2.12 of [14], $K(P) = K(A) = \{1\}$. Suppose $A \neq \{1\}$. Let $a \in P \setminus K(P)$ and $b \in K(A)$. Then by Lemma 10 of [15], we have $Cyc((a, b)) = Cyc(a) \times A$, which is not cyclic, a contradiction. Hence $A = \{1\}$. Therefore G is a p-group. By Theorem 14 of [15], there exists $H \trianglelefteq G$ such that H is cyclic or generalized quaternion and for all $x \in G \setminus H$ we have $o(x) = p$. If H is generalized quaternion, then by Theorem 8 of [15], $K(H) \neq \{1\}$. Let $y \in K(H) \setminus \{1\}$. Then $H \subseteq Cyc(y)$, which is a contradiction since $Cyc(y)$ is cyclic. Hence H is cyclic.

Conversely, by Theorem 8 of [15], we have $K(G) = \{1\}$. Suppose $h \in H \setminus \{1\}$. Clearly $H \subseteq Cyc(h)$. Suppose $H \subsetneq Cyc(h)$. Let $g \in Cyc(h) \setminus H$. Then $\langle g, h\rangle$ is cyclic and so $\langle g, h\rangle$ has exactly one subgroup of order p, namely $\langle g\rangle$. But $p \mid |\langle h\rangle|$ and so $\langle h\rangle$ has a subgroup of order p. Hence $\langle g\rangle \subseteq \langle h\rangle \subseteq H$, which is a contradiction. Hence $H = Cyc(h)$.

Again, since $o(x) = p$ for all $x \in G \setminus H$, therefore $Cyc(x) = \langle x\rangle$ for all $x \in G \setminus H$. Hence G is a C-tidy group with $K(G) = \{1\}$. □

Corollary 3.2. *G is an abelian C-tidy group with* $K(G) = \{1\}$ *if and only if* $G \cong C_p^k$ *for some prime p and some integer* $k > 1$.

Proof. By Theorem 3.1, G is a p-group, where p is a prime and by Theorem 11 of [15], we have $G \cong C_p^k$ for some integer $k > 1$. Converse is trivial. □

Using Proposition 2.2, we now prove the following theorem.

Theorem 3.3. *The groups* $G = PGL(2, p^h)$ *and* $G = PSL(2, p^h)$ *are C-tidy with* $K(G) = \{1\}$ *for every prime* p *and* $h \geq 1$.

Proof. We have $PSL(2, 2) \cong PGL(2, 2) \cong S_3$, $PSL(2, 3) \cong A_4$ and $PGL(2, 3) \cong S_4$ and one can easily observe that S_3, A_4 and S_4 are C-tidy groups with $K(S_3) = K(A_4) = K(S_4) = \{1\}$. Suppose $p^h \geq 4$. If $G = PSL(2, p^h)$ or $G = PGL(2, p^h)$, then by Proposition 2.4 of [4] and Proposition 3.21 of [1], G contains subgroups P, A and B such that P is elementary abelian p group, A, B are cyclic and $\Pi = \{P^x, A^x, B^x | x \in G\}$ is a partition for G. Again, we have $Cyc_{P^x}(y) = \langle y \rangle$ for any $y \in P^x \setminus \{1\}$. Hence by Proposition 2.2, G is C-tidy and it is easy to see that $K(G) = \{1\}$. □

For a group G and any $g, x \in G$, it is an easy exercise to show that $C(gxg^{-1}) = gC(x)g^{-1}$. The following proposition is an analog to this result.

Proposition 3.4. *Let* G *be a group. Then* $Cyc(gxg^{-1}) = gCyc(x)g^{-1}$ *for any* $g, x \in G$.

Proof. Suppose $a \in Cyc(x)$. Then $\langle a, x \rangle$ is cyclic and so $g\langle a, x \rangle g^{-1}$ is cyclic. But $g\langle a, x \rangle g^{-1} = \langle gag^{-1}, gxg^{-1} \rangle$. Hence $gag^{-1} \in Cyc(gxg^{-1})$.

Conversely, suppose $a \in Cyc(gxg^{-1})$. Then $\langle a, gxg^{-1} \rangle$ is cyclic. Therefore $\langle g^{-1}ag, x \rangle$ is cyclic. Hence $g^{-1}ag \in Cyc(x)$ and so $a \in gCyc(x)g^{-1}$. □

Theorem 3.5. *Let* G *be a C-tidy group with* $K(G) = \{1\}$. *If* G *is non-solvable, then* $G \cong PGL(2, p^h)$, p *odd or* $G \cong PSL(2, p^h)$ *for some prime* p *and* $h \geq 1$.

Proof. Let Π be the collection of all proper cyclicizers of G. By Proposition 2.1 and Proposition 3.4, Π is a normal non-trivial partition of G in which all components are nilpotent. Therefore by Suzuki [18], [19], we have $G \cong PGL(2, p^h)$, p odd or $G \cong PSL(2, p^h)$ for some prime p and $h \geq 1$ or the Suzuki group $Sz(2^{2m+1}), m > 0$. But by Theorem 2.6, $Sz(2^{2m+1}), m > 0$ is not tidy. Hence $G \cong PGL(2, p^h)$, p odd or $G \cong PSL(2, p^h)$ for some prime p and $h \geq 1$. □

The following theorem classifies all C-tidy groups with $K(G) = \{1\}$ in which the proper cyclicizers have equal order.

Theorem 3.6. *Let* G *be a group. Then* G *is C-tidy,* $K(G) = \{1\}$ *and* $|Cyc(x)| = |Cyc(y)|$ *for all* $x, y \in G \setminus \{1\}$ *if and only if* G *is a non-cyclic* p*-group of exponent* p*, for some prime* p.

Proof. Let Π be the collection of all proper cyclicizers of G. By Proposition 2.1, Π is a non-trivial partition of G in which all components of Π have same order. Therefore by [9], G is a p-group of exponent p, for some prime p. Again, since $K(G) = \{1\}$, therefore G is non-cyclic.

Conversely, suppose G is a non-cyclic p-group of exponent p, for some prime p. Then $Cyc(x) = \langle x \rangle$ for any $x \in G \setminus \{1\}$ and so $|Cyc(x)| = |Cyc(y)|$ for all $x, y \in G \setminus \{1\}$ and $K(G) = \{1\}$. □

Theorem 3.7. *Let* G *be a C-tidy group with* $K(G) = \{1\}$. *If* $Cyc(x)$ *is subnormal for all* $x \in G \setminus \{1\}$, *then* G *is a* p*-group for some prime* p.

Proof. Let Π be the collection of all proper cyclicizers of G. By Proposition 2.1, Π is a non-trivial partition of G. Now, the result follows from a result of Kegel (see p. 574 of [19], [12]). $\square$

Theorem 3.8. *Let G be a group. Then G is C-tidy, $K(G) = \{1\}$ and $Cyc(x) \trianglelefteq G$ for all $x \in G \setminus \{1\}$ if and only if $G \cong C_p^k$ for some prime p and some integer $k > 1$.*

Proof. Let Π be the collection of all proper cyclicizers of G. By Proposition 2.1, Π is a non-trivial partition of G such that $Cyc(x)Cyc(y) = Cyc(y)Cyc(x)$ for all $x, y \in G \setminus \{1\}$. Therefore by Theorem 8 of [9], $G \cong C_p^k$ for some prime p and some integer $k > 1$. Converse is trivial. $\square$

It is easy to see that if G is a C-tidy group then any proper subgroup of G is also C-tidy. For C-tidy groups with trivial center, we have the following result.

Theorem 3.9. *Let G be a group with trivial center. Then G is C-tidy if and only if H is cyclic or C-tidy with $K(H) = \{1\}$ for any $H \lneq G$.*

Proof. Suppose H is cyclic or C-tidy with $K(H) = \{1\}$ for any $H \lneq G$. Then by Proposition 2.3, H is completely decomposable. Hence by Kontorovich (p. 573 of [19]), G is completely decomposable and by Proposition 2.3, G is C-tidy.

Conversely, suppose H is a non-cyclic proper subgroup of G. Clearly H is C-tidy. If $x \in K(H) \setminus \{1\}$, then $H \subseteq Cyc(x)$, which forces H to be cyclic, a contradiction. $\square$

The following theorem classifies all C-tidy groups G with $K(G) = \{1\}$ having non-trivial center. Recall that the fitting subgroup $F(G)$ of a group G is the product of all nilpotent normal subgroups of G and it is the largest nilpotent normal subgroup of G.

Theorem 3.10. *Let G be a group with non-trivial center. Then G is C-tidy with $K(G) = \{1\}$ if and only if it satisfies one of the following conditions:*

(1) *G is a non-cyclic p-group with a cyclic normal subgroup H such that $o(x) = p$ for all $x \in G \setminus H$, where p is a prime,*
(2) *G is a non p-group with a cyclic normal subgroup H of prime index such that $|Cyc(x)| = p$ for all $x \in G \setminus H$, where p is a prime.*

Proof. If G is a p-group for some prime p, then (1) follows from Theorem 3.1. Suppose G is not a p-group, where p is a prime. Let Π be the collection of all proper cyclicizers of G. By Proposition 2.1, Π is a non-trivial partition of G. Now, by a result of Kegel [12], the fitting subgroup $F(G)$ is cyclic normal of prime index p. Therefore $F(G) \subseteq Cyc(y) \subsetneq G$ for any $y \in F(G) \setminus K(G)$ and so $F(G) = Cyc(y)$. Let $x \in G \setminus F(G)$. By Proposition 2.1, $Cyc(x) \cap Cyc(y) = K(G) = \{1\}$. Now, $G = Cyc(x)Cyc(y)$ and so $|G| = |Cyc(x)||Cyc(y)|$. Therefore $|Cyc(x)| = p$. Hence (2) follows.

Conversely, suppose the condition (1) holds. Then by Theorem 3.1, G is C-tidy with $K(G) = \{1\}$. Next, suppose H is a cyclic normal subgroup of G of prime index and $|Cyc(x)| = p$ for all $x \in G \setminus H$, where p is a prime. Then $H \subseteq Cyc(y) \subsetneq G$ for any $y \in H \setminus K(G)$ and so $Cyc(y) = H$. Hence G is C-tidy with $K(G) = \{1\}$. $\square$

We now classify Frobenius C-tidy groups.

Theorem 3.11. *Let G be a Frobenius group with Frobenius kernel K and p be a prime. Then G is C-tidy if and only if K is cyclic or K is a p-group with a cyclic normal subgroup N such that $o(x) = p$ for all $x \in K \setminus N$ and $C(y)$ is cyclic for all $y \in G \setminus K$.*

Proof. Suppose G is a C-tidy Frobenius group with Frobenius kernel K. Then K is a nilpotent C-tidy group. Suppose K is not cyclic. Then $K(K) = \{1\}$, and by Theorem 3.1, K is a p-group with a cyclic normal subgroup N such that $o(x) = p$ for all $x \in K \setminus N$, p being a prime.

Next, suppose $y \in G \setminus K$. Then $y \in H^g$ for some $g \in K$ where H^g is a component of the Frobenius partition. Note that H^g is a Frobenius complement and so by Lemma 3 , p. 273 of [5], every sylow subgroup of H^g is either cyclic or generalized quaternion. Therefore by Theorem 6 of [16] and Problem 7.1 of [10], $Cyc_{H^g}(y) = C_{H^g}(y) = C_G(y) = C(y)$. Hence $C(y)$ is cyclic. Converse follows from Theorem 3.1. □

The following theorem classifies Hughes-Thompson type C-tidy groups with $K(G) = \{1\}$.

Theorem 3.12. *Let G be a group of Hughes-Thompson type. Then G is C-tidy with $K(G) = \{1\}$ if and only if $H_p(G)$ is cyclic normal or $H_p(G)$ is a normal q-group with a cyclic normal subgroup N such that $o(h) = q$ for all $h \in H_p(G) \setminus N$ and $|Cyc(x)| = p$ for all $x \in G \setminus H_p(G)$, where p, q are distinct primes.*

Proof. We have $H_p(G) \neq G$ for some prime p. By a theorem of Hughes-Thompson [8], $|G : H_p(G)| = p$ and by a theorem of Kegel [11], $H_p(G)$ is nilpotent.

Now, suppose $H_p(G)$ is cyclic. Then $H_p(G) = \langle a \rangle$ for some $a \in H_p(G)$. Let $g \in G$. Then $gH_p(G)g^{-1} = g\langle a \rangle g^{-1} = \langle gag^{-1} \rangle$. Therefore $o(gag^{-1}) \neq p$ and so $gag^{-1} \in H_p(G)$. Hence $gH_p(G)g^{-1} = H_p(G)$ and $H_p(G) \trianglelefteq G$. Again, $H_p(G) = Cyc(y)$ for any $y \in H_p(G) \setminus \{1\}$. Therefore for any $x \in G \setminus H_p(G)$, we have $Cyc(x) = \langle x \rangle$ and so $|Cyc(x)| = p$.

Next, suppose $H_p(G)$ is not cyclic. Then $K(H_p(G)) = \{1\}$ and by Theorem 3.1, $H_p(G)$ is a q-group for some prime q with a cyclic normal subgroup N such that $o(h) = q$ for all $h \in H_p(G) \setminus N$. Again, if $q = p$, then G will be a p-group, which is a contradiction. Hence $q \neq p$. Clearly $H_p(G)$ is the sylow q-subgroup of G and so $H_p(G) \trianglelefteq G$. Now, suppose $x \in G \setminus H_p(G)$ and $b \in Cyc(x)$. If $b \in H_p(G) \setminus \{1\}$, then $o(bx) \neq p$ and so $bx \in H_p(G)$. Therefore $x = b^{-1}bx \in H_p(G)$, which is a contradiction. Hence $Cyc(x) = \langle x \rangle$ and so $|Cyc(x)| = p$.

Conversely, suppose $H_p(G)$ is cyclic and $|Cyc(x)| = p$ for all $x \in G \setminus H_p(G)$ where p is a prime. Then G is C-tidy with $K(G) = \{1\}$. Next, suppose $H_p(G)$ is a q-group with a cyclic normal subgroup N such that $o(h) = q$ for all $h \in H_p(G) \setminus N$ and $|Cyc(x)| = p$ for all $x \in G \setminus H_p(G)$, where p, q are distinct primes. Let $z \in H_p(G) \setminus \{1\}$. Then $Cyc_G(z) = Cyc_{H_p(G)}(z)$. Now, the result follows by Theorem 3.1. □

Finally, combining Proposition 2.1, Theorem 2.6, Theorem 2.7, Theorem 3.1, Theorem 3.3, Theorem 3.5, Theorem 3.11 and Theorem 3.12, we get the classification theorem for C-tidy groups with $K(G) = \{1\}$ as follows.

Theorem 3.13. *A group G is C-tidy with $K(G) = \{1\}$ if and only if it satisfies one of the following conditions:*

(1) *G is a non-cyclic p-group with a cyclic normal subgroup H such that $o(x) = p$ for all $x \in G \setminus H$, p being a prime,*
(2) *G is a Frobenius group in which the Frobenius kernel K is cyclic or K is a p-group with a cyclic normal subgroup N such that $o(x) = p$ for all $x \in K \setminus N$ and $C(y)$ is cyclic for all $y \in G \setminus K$, p being a prime,*
(3) *G is of Hughes-Thompson type in which $H_p(G)$ is cyclic normal or $H_p(G)$ is a normal q-group with a cyclic normal subgroup N such that $o(h) = q$ for all $h \in H_p(G) \setminus N$ and $|Cyc(x)| = p$ for all $x \in G \setminus H_p(G)$, p, q being distinct primes,*
(4) *$G \cong PGL(2, p^h)$, p being an odd prime, $h \geq 1$,*
(5) *$G \cong PSL(2, p^h)$, p being a prime, $h \geq 1$.*

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