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A NEW CHARACTERIZATION OF REE GROUP $^2G_2(q)$ BY THE ORDER OF GROUP AND THE NUMBER OF ELEMENTS WITH THE SAME ORDER

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Abstract. In this paper, we prove that Ree group $^2G_2(q)$, where $q \pm \sqrt{3q}+1$ is a prime number can be uniquely determined by the order of group and the number of elements with the same order.

1. Introduction

Let G be a finite group, $\pi(G)$ be the set of prime divisors of order of G and $\pi_e(G)$ be the set of orders of elements in G. If $k \in \pi_e(G)$, then we denote the number of elements of order k in G by $\mathsf{m}_k(G)$ and the set of the number of elements with the same order in G by $\mathsf{nse}(G)$. In other word,

$$\mathsf{m}_k(G) = |\{g \in G \ : \ o(g) = k\}|,$$

$$\mathsf{nse}(G) = \{\mathsf{m}_k(G) \ : \ k \in \pi_e(G)\}.$$

Also we denote a Sylow p-subgroup of G by G_p and the number of Sylow p-subgroups of G by $n_p(G)$. The prime graph $\Gamma(G)$ of group G is a graph whose vertex set is $\pi(G)$, and two distinct vertices u and v are adjacent if and only if $uv \in \pi_e(G)$. Moreover, assume that $\Gamma(G)$ has $t(G)$ connected components π_i, for $i = 1, 2, \ldots, t(G)$. In the case where G is of even order, we always assume that $2 \in \pi_1$.

The characterization of groups by nse pertains to Thompson's problem ([15]) which Shi posed it in [24]. The first time, this type of characterization was studied by Shao and Shi. In [23], they proved that if S is a finite simple group with $|\pi(S)| = 4$, then S is characterizable by $nse(S)$ and $|S|$. Following this result, in [5]-[7], it is proved that Sporadic simple groups, Projective special linear groups $PSL_2(q)$ and Suzuki groups $Sz(q)$, where $q-1$ is a prime number can be uniquely determined by order of group and nse. Also, it has been investigated that some other groups can be characterized by

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nse (see [8]-[12],[14],[16]-[18],[20]-[22],[27]-[29]). Moreover, it is proved that the group $D_n(3)$ is characterizable by prime graph [2] and the group $D_4(4)$ is OD-characterization [19]. In this paper, we prove that Ree group ${}^2G_2(q)$, where $q \pm \sqrt{3q}+1$ is a prime number can be uniquely determined by the order of group and the number of elements with the same order. In fact, we prove the following theorem.

Main Theorem. Let G be a group with $|G| = |{}^2G_2(q)|$ and $\mathsf{nse}(G) = \mathsf{nse}({}^2G_2(q))$, where $q \pm \sqrt{3q}+1$ is a prime number. Then $G \cong {}^2G_2(q)$.

2. Notation and Preliminaries

Lemma 2.1. [4] *Let G be a Frobenius group of even order with kernel K and complement H. Then*

(a) *$t(G) = 2$, $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$;*
(b) *$|H|$ divides $|K| - 1$;*
(c) *K is nilpotent.*

Definition 2.2. *A group G is called a 2-Frobenius group if there is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/H and K are Frobenius groups with kernels K/H and H respectively.*

Lemma 2.3. [1] *Let G be a 2-Frobenius group of even order. Then*

(a) *$t(G) = 2$, $\pi(H) \cup \pi(G/K) = \pi_1$ and $\pi(K/H) = \pi_2$;*
(b) *G/K and K/H are cyclic groups satisfying $|G/K|$ divides $|\mathsf{Aut}(K/H)|$.*

Lemma 2.4. [26] *Let G be a finite group with $t(G) \geq 2$. Then one of the following statements holds:*

(a) *G is a Frobenius group;*
(b) *G is a 2-Frobenius group;*
(c) *G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-abelian simple group, H is a nilpotent group and $|G/K|$ divides $|\mathsf{Out}(K/H)|$.*

Lemma 2.5. [3] *Let G be a finite group and m be a positive integer dividing $|G|$. If $L_m(G) = \{g \in G \mid g^m = 1\}$, then $m \mid |L_m(G)|$.*

Lemma 2.6. *Let G be a finite group. Then for every $i \in \pi_e(G)$, $\varphi(i)$ divides $\mathsf{m}_i(G)$, and i divides $\sum_{j|i} \mathsf{m}_j(G)$. Moreover, if $i > 2$, then $\mathsf{m}_i(G)$ is even.*

Proof. By Lemma (2.5), the proof is straightforward. □

Lemma 2.7. [25] *Let G be a non-abelian simple group such that $(5, |G|) = 1$. Then G is isomorphic to one of the following groups:*

(a) *$A_n(q)$, $n = 1, 2$, $q \equiv \pm 2 \pmod 5$;*
(b) *$G_2(q)$, $q \equiv \pm 2 \pmod 5$;*
(c) *${}^2A_2(q)$, $q \equiv \pm 2 \pmod 5$;*
(d) *${}^3D_4(q)$, $q \equiv \pm 2 \pmod 5$;*
(e) *${}^2G_2(q)$, $q = 3^{2m+1}$, $m \geq 1$.*

Lemma 2.8. [30] *Let q, k, l be natural numbers. Then*

(a) $(q^k-1, q^l-1) = q^{(k,l)}-1$.

(b) $(q^k+1, q^l+1) = \begin{cases} q^{(k,l)}+1 & \text{if both } \frac{k}{(k,l)} \text{ and } \frac{l}{(k,l)} \text{ are odd,} \\ (2, q+1) & \text{otherwise.} \end{cases}$

(c) $(q^k-1, q^l+1) = \begin{cases} q^{(k,l)}+1 & \text{if } \frac{k}{(k,l)} \text{ is even and } \frac{l}{(k,l)} \text{ is odd,} \\ (2, q+1) & \text{otherwise.} \end{cases}$

In particular, for every $q \geq 2$ and $k \geq 1$, the inequality $(q^k-1, q^k+1) \leq 2$ holds.

Lemma 2.9. *Let G be a Ree group ${}^2G_2(q)$, where $p = q \pm \sqrt{3q}+1$ is a prime number. Then $\mathsf{m}_p(G) = (p-1)|G|/(6p)$ and for every $i \in \pi_e(G) - \{1, p\}$, p divides $\mathsf{m}_i(G)$.*

Proof. Since $|G_p| = p$, we deduce that G_p is a cyclic group of order p. Thus $\mathsf{m}_p(G) = \varphi(p)n_p(G) = (p-1)n_p(G)$. Now it is enough to show $n_p(G) = |G|/(6p)$. By [26], p is an isolated vertex of $\Gamma(G)$. Hence $|C_G(G_p)| = p$ and $|N_G(G_p)| = \alpha p$ for a natural number α. We know that $N_G(G_p)/C_G(G_p)$ embed in $\mathsf{Aut}(G_p)$, which implies $\alpha \mid (p-1)$. Furthermore, by Sylow's Theorem, $n_p(G) = |G : N_G(G_p)|$ and $n_p(G) \equiv 1 \pmod p$. Therefore p divides $|G|/(\alpha p) - 1$. Thus $q \pm \sqrt{3q}+1$ divides $(q^5 \mp 2\sqrt{3q}q^4 + 6q^4 \mp 4\sqrt{3q}q^3 + 5q^3 - 6q^2 \pm 6\sqrt{3q}q - 12q \pm 6\sqrt{3q} - 6)(q \pm \sqrt{3q}+1) + (6-\alpha)$. It follows that $p \mid (6-\alpha)$ and since $\alpha \mid (p-1)$, we obtain that $\alpha = 6$.

Let $i \in \pi_e(G) - \{1, p\}$. Since p is an isolated vertex of $\Gamma(G)$, we conclude that $p \nmid i$ and $pi \notin \pi_e(G)$. Thus G_p acts fixed point freely on the set of elements of order i by conjugation and hence $|G_p| \mid \mathsf{m}_i(G)$. So we conclude that $p \mid \mathsf{m}_i(G)$. □

3. Proof of the Main Theorem

In this section, we prove the main theorem by the following lemmas. We denote the Ree group ${}^2G_2(q)$, where $q = 3^{2m+1}$, $m \geq 1$ by R and prime number $q \pm \sqrt{3q}+1$ by p. Recall that G is a group with $|G| = |R|$ and $\mathsf{nse}(G) = \mathsf{nse}(R)$.

Lemma 3.1. *$\mathsf{m}_2(G) = \mathsf{m}_2(R)$, $\mathsf{m}_p(G) = \mathsf{m}_p(R)$, $n_p(G) = n_p(R)$, p is an isolated vertex of $\Gamma(G)$ and $p \mid \mathsf{m}_k(G)$ for every $k \in \pi_e(G) - \{1, p\}$.*

Proof. By Lemma (2.6), for every $1 \neq i \in \pi_e(G)$, $i = 2$ if and only if $\mathsf{m}_i(G)$ is odd. Thus we deduce that $\mathsf{m}_2(G) = \mathsf{m}_2(R)$. According to Lemma (2.6), $(\mathsf{m}_p(G), p) = 1$. Thus $p \nmid \mathsf{m}_p(G)$ and hence Lemma (2.9) implies that $\mathsf{m}_p(G) \in \{\mathsf{m}_1(R), \mathsf{m}_2(R), \mathsf{m}_p(R)\}$. Moreover, $\mathsf{m}_p(G)$ is even, so we conclude that $\mathsf{m}_p(G) = \mathsf{m}_p(R)$. Since G_p and R_p are cyclic groups of order p and $\mathsf{m}_p(G) = \mathsf{m}_p(R)$, we deduce that $\mathsf{m}_p(G) = \varphi(p)n_p(G) = \varphi(p)n_p(R) = \mathsf{m}_p(R)$, so $n_p(G) = n_p(R)$.

Now we prove that p is an isolated vertex of Γ(G). Assume the contrary. Then there is $t \in \pi(G) - \{p\}$ such that $tp \in \pi_e(G)$. So $\mathsf{m}_{tp}(G) = \varphi(tp)n_p(G)k$, where k is the number of cyclic subgroups of order t in $C_G(G_p)$ and since $n_p(G) = n_p(R)$, it follows that $\mathsf{m}_{tp}(G) = (t-1)(p-1)|R|k/(6p)$. If $\mathsf{m}_{tp}(G) = \mathsf{m}_p(R)$, then $t = 2$ and $k = 1$. Furthermore, Lemma (2.5) yields $p \mid (\mathsf{m}_2(G) + \mathsf{m}_{2p}(G))$ and since $\mathsf{m}_2(G) = \mathsf{m}_2(R)$ and $p \mid \mathsf{m}_2(R)$, we have $p \mid \mathsf{m}_{2p}(G)$ which is a contradiction. So Lemma (2.9) implies that $p \mid \mathsf{m}_{tp}(G)$.

Hence $p \mid (t-1)$ and since $\mathsf{m}_{tp}(G) < |G|$, we deduce that $p - 1 \leq 6$. But this is impossible because $p = q \pm \sqrt{3q} + 1$ and $q = 3^{2m+1}$, $m \geq 1$.

Let $k \in \pi_e(G) - \{1, p\}$. Since p is an isolated vertex of $\Gamma(G)$, $p \nmid k$ and $pk \notin \pi_e(G)$. Thus G_p acts fixed point freely on the set of elements of order k by conjugation and hence $|G_p| \mid \mathsf{m}_k(G)$. So we conclude that $p \mid \mathsf{m}_k(G)$. □

Lemma 3.2. *The group G is neither a Frobenius group nor a 2-Frobenius group.*

Proof. Let G be a Frobenius group with kernel K and complement H. Then by Lemma (2.1), $t(G) = 2$ and $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$ and $|H|$ divides $|K| - 1$. Now by Lemma (3.1), p is an isolated vertex of $\Gamma(G)$. Thus we deduce that (i) $|H| = p$ and $|K| = |G|/p$ or (ii) $|H| = |G|/p$ and $|K| = p$. Since $|H|$ divides $|K| - 1$, we conclude that the last case can not occur. So $|H| = p$ and $|K| = |G|/p$, hence $(q \pm \sqrt{3q} + 1) \mid \Big((q^5 \mp 2\sqrt{3q}q^4 + 6q^4 \mp 4\sqrt{3q}q^3 + 5q^3 - 6q^2 \pm 6\sqrt{3q}q - 12q \pm 6\sqrt{3q} - 6)(q \pm \sqrt{3q} + 1) + 5\Big)$. Thus $p \mid 5$ which is impossible.

We now show that G is not a 2-Frobenius group. Let G be a 2-Frobenius group. Then G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/H and K are Frobenius groups by kernels K/H and H respectively. Set $|G/K| = x$. Since p is an isolated vertex of $\Gamma(G)$, we have $|K/H| = p$ and $|H| = |G|/(xp)$. By Lemma (2.3), $|G/K|$ divides $|\mathsf{Aut}(K/H)|$. Thus $x \mid (p-1)$ and since by Lemma (2.8), $(\frac{1}{4}(q+1), p-1) = 1$, $\frac{1}{4}(q+1) \mid |H|$. Therefore $H_t \rtimes K/H$ is a Frobenius group with kernel H_t and complement K/H, where $t = \frac{1}{4}(q+1)$. So $|K/H|$ divides $|H_t| - 1$. It implies that $(q \pm \sqrt{3q} + 1) \mid (\frac{1}{4}(q+1) - 1)$, but this is a contradiction. □

Lemma 3.3. *The group G is isomorphic to the group R.*

Proof. By Lemma (3.1), p is an isolated vertex of $\Gamma(G)$. Thus $t(G) > 1$ and G satisfies one of the cases of Lemma (2.4). Now Lemma (3.2) implies that G is neither a Frobenius group nor a 2-Frobenius group. Thus only the case (c) of Lemma (2.4) occure. So G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-abelian simple group. Since p is an isolated vertex of $\Gamma(G)$, we have $p \mid |K/H|$. On the other hand, we know that $5 \nmid |G|$. Thus K/H is isomorphic to one of the groups in Lemma (2.7).

(1) If $K/H \cong G_2(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [13, 26], $p = q'^2 \mp q' + 1$. Thus $3^{m+1}(3^m \pm 1) = q'(q' \mp 1)$ and since $(q', q' \mp 1) = 1$, we have (i) $3^{m+1} = q'$ and $3^m \pm 1 = q' \mp 1$ or (ii) $3^{m+1} \mid (q' \pm 1)$ and $q' \mid (3^m \mp 1)$ which is a contradiction. The same as above procedure, we can prove that $K/H \ncong {}^3D_4(q')$, where $q' \equiv \pm 2 \pmod 5$.

(2) If $K/H \cong {}^2A_2(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [13, 26], $p = (q'^2 - q' + 1)/(3, q' + 1)$. If $(3, q' + 1) = 1$, then similar to part (1) we deduce a contradiction. Now let $(3, q' + 1) = 3$. Then $3^{m+2}(3^m \pm 1) = (q' - 2)(q' + 1)$. Thus $3^m(3^m \pm 1) = \frac{1}{3}(q' - 2)\frac{1}{3}(q' + 1)$. It follows that (i) $3^m \mid \frac{1}{3}(q' - 2)$ and $\frac{1}{3}(q' + 1) \mid (3^m \pm 1)$ or (ii) $3^m \mid \frac{1}{3}(q' + 1)$ and $\frac{1}{3}(q' - 2) \mid (3^m \pm 1)$. If the case (i) occure, then $q' = \sqrt{3q} + 2$. On the other hand, we have $|K/H| = \frac{1}{3}q'^3(q'^2 - 1)(q'^3 + 1)$ and $|G| = q^3(q^3 + 1)(q - 1)$ and since $(3, q'+1) = 3$, we deduce that $(3, q') = 1$. Thus $q'^3 \mid (q^3+1)$ or $q'^3 \mid (q-1)/2$. So $(\sqrt{3q}+2)^3 \mid (q^3+1)$ or $(\sqrt{3q}+2)^3 \mid (q-1)$ which is a contradiction. If the case (ii) occure, then $q' = \sqrt{3q}-1$. On the other hand, we have $|K/H| = \frac{1}{3}q'^3(q'^2-1)(q'^3+1)$ and $|G| = q^3(q^3+1)(q-1)$ and since $(3, q'+1) = 3$, we deduce that $(3, q') = 1$. Thus $q'^3 \mid (q^3+1)$ or $q'^3 \mid (q-1)/2$. So $(\sqrt{3q}-1)^3 \mid (q^3+1)$ or $(\sqrt{3q}-1)^3 \mid (q-1)$ which is a contradiction. Similarly, we can prove that $K/H \ncong A_n(q')$, where $n = 1, 2$ and $q' \equiv \pm 2 \pmod 5$.

So we deduce that $K/H \cong {}^2G_2(q')$, where $q' = 3^{2m'+1}$. Now since $\pi_2({}^2G_2(q')) = q' \pm \sqrt{3q'} + 1$, we conclude that, $p = q' \pm \sqrt{3q'} + 1$. Thus $q \pm \sqrt{3q} = q' \pm \sqrt{3q'}$ and hence $m = m'$ and $q = q'$ and $K/H \cong R$. Now since $|K/H| = |R| = |G|$ and $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$, we have $H = 1$, $G = K \cong R$ and the proof is completed. □

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