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TRANSITIVE t-DESIGNS AND STRONGLY REGULAR GRAPHS CONSTRUCTED FROM LINEAR GROUPS $L(2,q)$, $q \leq 23$

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Abstract. In this paper we construct transitive t-designs from the linear groups $L(2,q), q \leq 23$. Thereby we classify t-designs, $t \geq 2$, admitting a transitive action of the linear groups $L(2,q), q \leq 23$, up to 35 points and obtained numerous transitive designs, for $36 \leq v \leq 55$. In many cases we proved the existence of t-designs with certain parameter sets. Among others we constructed t-designs with parameters 2-$(55,10,4)$, 3-$(24,11,495)$, 3-$(24,12,5m), m \in \{11,12,22,33,44,66,132\}$. Furthermore, we constructed strongly regular graphs admitting a transitive action of the linear groups $L(2,q), q \leq 23$.

1. Introduction

We assume that the reader is familiar with the basic facts of group theory, design theory and theory of strongly regular graphs. We refer the reader to [2, 19], [7, 17] and [2, 3, 19] for related background materials in design theory, group theory and theory of strongly regular graphs, respectively.

An incidence structure is an ordered triple $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ where $\mathcal{P}$ and $\mathcal{B}$ are non-empty disjoint sets and $\mathcal{I} \subseteq \mathcal{P} \times \mathcal{B}$. The elements of the set $\mathcal{P}$ are called points, the elements of the set $\mathcal{B}$ are called blocks and $\mathcal{I}$ is called an incidence relation. An incidence structure that does not contain repeated blocks (blocks incident with the same set of points) is called simple. If $|\mathcal{P}| = |\mathcal{B}|$, then the incidence structure is called symmetric. The incidence matrix of an incidence structure is a $v \times b$ matrix $[m_{ij}]$ where v and b are the numbers of points and blocks respectively, such that $m_{ij} = 1$ if the point P_i and the block x_j are incident, and $m_{ij} = 0$ otherwise. An isomorphism from one incidence structure to another is a bijective mapping of points to points and blocks to blocks which preserves incidence. An isomorphism from an

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incidence structure $\mathcal{D}$ onto itself is called an automorphism of $\mathcal{D}$. The set of all automorphisms forms a group called the full automorphism group of $\mathcal{D}$ and is denoted by $Aut(\mathcal{D})$.

A t-(v,k,λ) design is a finite incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ satisfying the following requirements:

(1) $|\mathcal{P}| = v$,
(2) every element of $\mathcal{B}$ is incident with exactly k elements of $\mathcal{P}$,
(3) every t elements of $\mathcal{P}$ are incident with exactly λ elements of $\mathcal{B}$.

If $\mathcal{D}$ is a t-design, then it is also an s-design, for $1 \leq s \leq t-1$. A 2-(v,k,λ) design is called a block design. We say that a t-(v,k,λ) design $\mathcal{D}$ is a quasi-symmetric design with intersection numbers x and y $(x < y)$ if any two blocks of $\mathcal{D}$ intersect in either x or y points.

A graph is regular if all the vertices have the same degree; a regular graph is strongly regular of type (v,k,λ,μ) if it has v vertices, degree k, and if any two adjacent vertices are together adjacent to λ vertices, while any two non-adjacent vertices are together adjacent to μ vertices. A strongly regular graph of type (v,k,λ,μ) is usually denoted by $\mathrm{SRG}(v,k,\lambda,\mu)$.

In this paper we consider t-designs and strongly regular graphs constructed from the linear groups $L(2,q), q \leq 23$.

The group $L(2,q)$ acts 3-transitively on subsets of size 3 of the projective line if $q \equiv 3 (\mathrm{mod}\ 4)$. Thereby, unions of orbits of a subgroup of $L(2,q)$ produce simple 3-designs under the action of $L(2,q)$. However, if $q \equiv 1 (\mathrm{mod}\ 4)$, the action is not 3-transitive so we do not have to obtain 3-designs. In [6], Cameron, Maimani, Omidi and Tayfeh-Rezaie determined all 3-designs admitting an automorphism group isomorphic to $L(2,q)$ with block size not congruent to 0 and 1 modulo p, where $q = p^n$ is a prime power congruent to 3 modulo 4. Further information on the research regarding linear groups $L(2,q)$, where $q \equiv 1 (\mathrm{mod}\ 4)$, can be found in [1], but the whole enumeration of t-designs admitting a transitive action of the groups $L(2,q), q \leq 23$, have not been described so far.

Using the method introduced in [10], we classify t-designs, $t \geq 2$, admitting a transitive action of the linear groups $L(2,q), q \leq 23$, up to 35 points. Additionally, we obtained numerous transitive designs, for $36 \leq v \leq 55$. All the designs obtained in this paper are simple. A number of the designs obtained in this paper including 2-$(55,10,4)$, 3-$(24,11,495)$, 3-$(24,12,5m), m \in \{11,12,22,33,44,66,132\}$ are the first known examples of the designs with these parameters. Further, we construct strongly regular graphs from the linear groups $L(2,q), q \leq 23$.

Generators of the linear groups $L(2,q), q \leq 23$ are available via the Internet:
http://brauer.maths.qmul.ac.uk/Atlas/. All the t-designs are obtained by using programmes written for Magma [5].

The paper is organized as follows: in Section 2 we briefly describe the method of construction of transitive designs and graphs used in this paper, and in Section 3 we describe t-designs and strongly regular graphs constructed under the action of the linear groups $L(2,q), q \leq 23$.

2. Designs and graphs constructed from groups

We say that an incidence structure $\mathcal{I}$ is transitive if an automorphism group of $\mathcal{I}$ acts transitively on points and blocks. An incidence structure $\mathcal{I}$ is called primitive if an automorphism group acts primitively on points and blocks. Further, we say that graph Γ is transitive (primitive) if an automorphism group acts transitively (primitively) on the set of vertices of the graph Γ.

The construction of primitive symmetric 1-designs and regular graphs for which the stabilizer of a point and the stabilizer of a block are conjugate is presented in [11], [12] and [13]. The generalization, *i.e.* the method for constructing not necessarily symmetric but still primitive 1-designs, is presented in [8] and [9]. In [10], a construction of not necessarily primitive, but still transitive block designs is presented.

Theorem 2.1 ([10])**.** *Let G be a finite permutation group acting transitively on the sets Ω_1 and Ω_2 of size m and n, respectively. Let $\alpha \in \Omega_1$ and $\Delta_2 = \bigcup_{i=1}^{s} \delta_i G_\alpha$, where $G_\alpha = \{g \in G \mid \alpha g = \alpha\}$ is the stabilizer of α and $\delta_1, \dots, \delta_s \in \Omega_2$ are representatives of distinct G_α-orbits on Ω_2. If $\Delta_2 \neq \Omega_2$ and*

$$\mathcal{B} = \{\Delta_2 g : g \in G\},$$

then $\mathcal{D}(G, \alpha, \delta_1, \dots, \delta_s) = (\Omega_2, \mathcal{B})$ is a 1-$(n, |\Delta_2|, \frac{|G_\alpha|}{|G_{\Delta_2}|} \sum_{i=1}^{s} |\alpha G_{\delta_i}|)$ design with $\frac{m \cdot |G_\alpha|}{|G_{\Delta_2}|}$ blocks. The group $H \cong G / \bigcap_{x \in \Omega_2} G_x$ acts as an automorphism group on $(\Omega_2, \mathcal{B})$, transitively on points and blocks of the design.

If $\Delta_2 = \Omega_2$ then the set $\mathcal{B}$ consists of one block, and $\mathcal{D}(G, \alpha, \delta_1, ..., \delta_s)$ is a design with parameters 1-$(n, n, 1)$.

If the group G acts t-transitively on the set Ω_2, then the obtained design $(\Omega_2, \mathcal{B})$ is a t-design (see [10]). However, if G does not act t-transitively on the set Ω_2, the obtained design $(\Omega_2, \mathcal{B})$ may still be a t-design.

In this paper we describe construction of t-designs, for $t \geq 2$, from linear groups $L(2, q), q \leq 23$, obtained by using Theorem 2.1.

Corollary 2.2. *If $\Omega_1 = \Omega_2$ and Δ_2 is a union of self-paired and mutually paired orbits of G_α, then the design $\mathcal{D}(G, \alpha, \delta_1, \dots, \delta_s)$ is a symmetric self-dual design and the incidence matrix of that design is the adjacency matrix of a $|\Delta_2|$-regular graph.*

If $G_\alpha = G_{\Delta_2}$, we can interpret the design $(\Omega_2, \mathcal{B})$ from Theorem 2.1 in the following way:

- the point set is Ω_2,
- the block set is $\Omega_1 = \alpha G$,
- the block $\alpha g'$ is incident with the set of points $\{\delta_i g : g \in G_\alpha g', i = 1, \dots, s\} = \Delta_2 g'$.

If $G_\alpha \neq G_{\Delta_2}$, then the above described design with the block set Ω_1 has repeated blocks, and the corresponding simple design is the design $(\Omega_2, \mathcal{B})$ from Theorem 2.1, which has $\frac{m \cdot |G_\alpha|}{|G_{\Delta_2}|}$ blocks.

Note that the construction from Theorem 2.1 gives us 1-designs and regular graphs. Since every t-design, for $t \geq 1$, is also a 1-design, Theorem 2.1 gives us all designs which admit a transitive action of the group G on points and blocks. Using this method of construction we obtained all 1-designs and regular graphs admitting a transitive action of the linear groups $L(2,q), q \leq 23$ but we recorded only those 1-designs that are also t-designs, $t \geq 2$, and those regular graphs that are strongly regular.

3. Results

In this section we list all t-designs and strongly regular graphs that we constructed using the method described in Section 2. Each subsection contains the information about the linear group that we used, subgroups of the group and it's properties and the data about obtained t-designs. The last section contains information about constructed strongly regular graphs. For each constructed structure (t-design and strongly regular graph) we give the structure of the full automorphism group, the number of non-isomorphic t-designs and remarks about constructed t-design and SRG. In tables to follow, for designs indicated by $*$ we proved the existence in terms of parameters.

3.1. Transitive t-designs from $L(2,5)$.

It is well known (see [7]) that the group $L(2,5) \cong L(2,4) \cong A_5$. The linear group $L(2,5)$ is the simple group of order 60 and up to conjugation it has 9 subgroups. In Table 1 we give the list of all the subgroups, up to conjugation. We classify t-designs, $t \geq 2$, admitting transitive action of the group $L(2,5)$. The constructed designs are presented in Table 2.

TABLE 1. Subgroups of the group $L(2,5)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	60	No	S_3	6	10	Yes
Z_2	2	30	No	D_{10}	10	6	Yes
Z_3	3	20	No	A_4	12	5	Yes
E_4	4	15	No	A_5	60	1	No
Z_5	5	12	No				

TABLE 2. t-designs constructed from the group $L(2,5)$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(6,3,2)$	10	1	A_5
2-$(10,3,2)$	30	1	A_5
2-$(10,4,2)$	15	1	S_6
2-$(10,4,8)$	60	1	S_5
2-$(15,7,3)$	15	1	A_8
2-$(15,7,12)$	60	1	$GL(2,4):Z_2$
		2	$GL(2,4)$

Remark 3.1. *Transitive t-designs described in Table 2 were all previously known. For further information on quasi-symmetric* 2-$(10,4,2)$ *we refer the reader to* [18] *and for other designs we refer the reader to* [16].

3.2. **Transitive t-designs from $L(2,7)$.** The linear group $L(2,7)$ is the simple group of order 168 and up to conjugation it has 15 subgroups. It is well known (see [7]) that $L(2,7) \cong L(3,2)$. We classify t-designs, $t \geq 2$, $v < 56$ admitting transitive action of the group $L(2,7)$. In Table 3 we give the list of all the subgroups, up to conjugation, and in Table 4 we present the constructed designs.

Table 3. Subgroups of the group $L(2,7)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	168	No	D_8	8	21	No
Z_2	2	84	No	A_4	12	14	No
Z_3	3	56	No	A_4	12	14	No
E_4	4	42	No	$Z_7 : Z_3$	21	8	Yes
E_4	4	42	No	S_4	24	7	Yes
Z_4	4	42	No	S_4	24	7	Yes
S_3	6	28	No	$L(2,7)$	168	1	No
Z_7	7	24	No				

Table 4. t-designs constructed from the group $L(2,7)$, $v < 56$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(7,3,1)$	7	1	$L(2,7)$
2-$(7,3,4)$	28	1	$L(2,7)$
3-$(8,3,1)$	56	1	S_8
3-$(8,4,1)$	14	1	$E_8 : L(2,7)$
3-$(8,4,3)$	42	1	$L(2,7) : Z_2$
2-$(21,5,4)$	84	2	$L(2,7)$
2-$(28,9,16)^*$	168	85	$L(2,7)$
		5	$L(2,7)$
2-$(28,10,20)^*$	168	1	$E_8 : L(2,7)$
		8	$L(2,7) : Z_2$
		213	$L(2,7)$

Remark 3.2. *Designs with parameters* 2-$(28,9,16)$ *and* 2-$(28,10,20)$ *are not mentioned in the* [16] *since* $r > 41$. *Up to our best knowledge they have not been known before, so we proved the existence of the* 2-$(28,9,16)$ *and* 2-$(28,10,20)$. *All others transitive t-designs described in Table 4 were previously known (see* [14, 16] *). For further information on unique quasi-symmetric* 3-$(8,4,1)$ *design we refer the reader to* [18].

3.3. **Transitive t-designs from $L(2,8)$.** The linear group $L(2,8)$ is the simple group of order 504 and up to conjugation it has 12 subgroups, given in Table 5. We classify t-designs, $t \geq 2$, $v \leq 28$ admitting transitive action of the group $L(2,8)$ and the obtained designs are listed in Table 6. Additionally, we classify t-designs, $t \geq 2$, $v = 36$, $b \leq 252$ admitting transitive action of the group $L(2,8)$ and the obtained designs are listed in Table 7.

TABLE 5. Subgroups of the group $L(2,8)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	504	No	E_8	8	63	No
Z_2	2	252	No	Z_9	9	56	No
Z_3	3	168	No	D_{14}	14	36	Yes
E_4	4	126	No	D_{18}	18	28	Yes
S_3	6	84	No	$E_8 : Z_7$	56	9	Yes
Z_7	7	72	No	$L(2,8)$	504	1	No

TABLE 6. t-designs constructed from the group $L(2,8)$, $v \leq 28$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
4-$(9,4,1)$	126	1	S_9
3-$(9,3,1)$	84	1	S_9
2-$(28,3,4)^*$	504	1	$L(2,8):Z_3$
2-$(28,4,1)$	63	1	$L(2,8):Z_3$
2-$(28,4,8)^*$	504	1	$L(2,8):Z_3$
		1	$L(2,8)$
2-$(28,6,20)^*$	504	3	$L(2,8):Z_3$
		16	$L(2,8)$
2-$(28,7,14)^*$	252	1	$L(2,8)$
2-$(28,7,28)^*$	504	4	$L(2,8):Z_3$
		12	$L(2,8)$
2-$(28,9,48)^*$	504	12	$L(2,8):Z_3$
		158	$L(2,8)$
2-$(28,10,30)^*$	252	2	$L(2,8)$
2-$(28,10,60)^*$	504	11	$L(2,8):Z_3$
		290	$L(2,8)$
2-$(28,12,11)$	63	1	$L(2,8)$
		1	$S(6,2)$
2-$(28,12,44)^*$	252	1	$L(2,8)$
2-$(28,12,88)^*$	504	19	$L(2,8):Z_3$
		587	$L(2,8)$
2-$(28,13,52)^*$	252	2	$L(2,8)$
2-$(28,13,104)^*$	504	18	$L(2,8):Z_3$
		723	$L(2,8)$

TABLE 7. t-designs constructed from the group $L(2,8)$, $v = 36, b \leq 252$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(36,5,4)$	252	1	$L(2,8)$
2-$(36,6,2)$	84	1	$L(2,8):Z_3$
2-$(36,6,6)^*$	252	3	$L(2,8):Z_3$
2-$(36,10,9)$	126	1	$L(2,8):Z_3$
2-$(36,10,18)^*$	252	4	$L(2,8):Z_3$
		1	$L(2,8)$
2-$(36,11,22)^*$	252	4	$L(2,8):Z_3$
2-$(36,15,28)^*$	168	1	$L(2,8):Z_3$
		1	$L(2,8)$
2-$(36,16,12)$	63	1	$L(2,8):Z_3$
		1	$S(6,2)$
2-$(36,16,48)^*$	252	2	$L(2,8):Z_3$

Remark 3.3. *For $t = 3,4$ designs described in Table 6 were known before. For quasi-symmetric* 2-$(28,4,1)$*,* 2-$(28,12,11)$ *and* 2-$(36,16,12)$ *designs we refer the reader to [10, 16, 18]. Majority of 2-designs listed in Table 6 and Table 7 have $r > 41$.*

3.4. **Transitive t-designs from** $L(2,9)$**.** It is well known (see [7]) that $L(2,9) \cong S(4,2)' \cong A_6 \cong M'_{10}$. The linear group $L(2,9)$ is the simple group of order 360 and up to conjugation it has 22 subgroups, listed in Table 8. We classify t-designs, $t \geq 2$, $v \leq 30$ admitting transitive action of the group $L(2,8)$ and the obtained designs are listed in Table 9. Additionally, we classify t-designs, $t \geq 2$, $v = 36$, $b \leq 180$ and $v = 40$, $b \leq 180$ admitting transitive action of the group $L(2,9)$ and the obtained designs are listed in Table 10.

TABLE 8. Subgroups of the group $L(2,9)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	360	No	E_9	9	40	No
Z_2	2	180	No	D_{10}	10	36	No
Z_3	3	120	No	A_4	12	30	No
Z_3	3	120	No	A_4	12	30	No
E_4	4	90	No	$E_9:Z_2$	18	20	No
E_4	4	90	No	S_4	24	15	Yes
Z_4	4	90	No	S_4	24	15	Yes
Z_5	5	72	No	$E_9:Z_4$	36	10	Yes
S_3	6	60	No	A_5	60	6	Yes
S_3	6	60	No	A_5	60	6	Yes
D_8	8	45	No	$L(2,9)$	360	1	No

Table 9. t-designs constructed from the group $L(2,9)$, $v \leq 30$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-$(6,3,1)$	20	1	S_6
3-$(10,4,6)$	180	1	$A_6 : E_4$
3-$(10,5,3)$	36	1	$A_6.Z_2$
2-$(10,3,4)$	60	1	S_6
2-$(10,4,2)$	15	1	S_6
2-$(10,5,20)^*$	90	1	S_6
2-$(15,7,3)$	15	1	A_8
2-$(15,7,24)^*$	120	1	A_8
2-$(15,7,36)^*$	180	1	S_6
2-$(15,7,72)^*$	360	1	A_6
		2	S_6

Table 10. t-designs constructed from the group $L(2,9)$, $v = 36, 40, b \leq 180$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(36,8,4)$	90	1	S_6
2-$(36,15,12)$	72	1	A_6
		1	S_6
2-$(36,15,30)^*$	180	3	A_6
2-$(40,13,4)$	40	1	$L(4,3) : Z_2$

Remark 3.4. *For $t = 3$ designs described in Table 9 were known before.*

3.5. **Transitive t-designs from $L(2,11)$.** The linear group $L(2,11)$ is the simple group of order 660 and up to conjugation it has 16 subgroups, which are listed in Table 11. In Table 12 we give the information on the obtained designs.

We classify t-designs, $t \geq 2$, $v < 55$ admitting transitive action of the group $L(2,11)$ and the obtained designs are listed in Table 12. Additionally, we classify t-designs, $t \geq 2$, $v = 55$, $b \leq 330$, admitting transitive action of the group $L(2,11)$ and the obtained designs are listed in Table 13.

Table 11. Subgroups of the group $L(2,11)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	660	No	D_{10}	10	66	No
Z_2	2	330	No	Z_{11}	11	60	No
Z_3	3	220	No	A_4	12	55	No
E_4	4	165	No	D_{12}	12	55	Yes
Z_5	5	132	No	$Z_{11} : Z_5$	55	12	Yes
S_3	6	110	No	A_5	60	11	Yes
S_3	6	110	No	A_5	60	11	Yes
Z_6	6	110	No	$L(2,11)$	660	1	No

TABLE 12. t-designs constructed from the group $L(2,11)$, $v < 55$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(11,3,3)$	55	1	$L(2,11)$
2-$(11,3,6)$	110	1	$L(2,11)$
2-$(11,4,6)$	55	1	$L(2,11)$
2-$(11,4,12)$	110	1	$L(2,11)$
2-$(11,4,18)^*$	165	1	$L(2,11)$
2-$(11,5,2)$	11	1	$L(2,11)$
2-$(11,5,10)$	55	1	$L(2,11)$
2-$(11,5,12)$	66	1	$L(2,11)$
2-$(11,5,60)^*$	330	1	$L(2,11)$
5-$(12,6,1)$	132	1	M_{12}
3-$(12,3,1)$	220	1	S_{12}
3-$(12,4,3)$	165	1	$L(2,11):Z_2$
3-$(12,4,6)$	330	1	$L(2,11):Z_2$
3-$(12,5,6)$	132	1	$L(2,11):Z_2$
3-$(12,5,30)$	660	1	$L(2,11):Z_2$
3-$(12,6,10)$	110	1	$L(2,11):Z_2$
		1	$L(2,11)$
3-$(12,6,30)$	330	1	$L(2,11):Z_2$

TABLE 13. t-designs constructed from the group $L(2,11)$, $v = 55, b \leq 330$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(55,9,8)^*$	330	2	$L(2,11)$
2-$(55,10,4)^*$	132	2	$L(2,11)$
2-$(55,10,10)^*$	330	6	$L(2,11)$
2-$(55,18,34)^*$	330	1	$L(2,11):Z_2$
		29	$L(2,11)$
2-$(55,19,38)^*$	330	7	$L(2,11)$
2-$(55,27,78)^*$	330	1	$L(2,11):Z_2$
		48	$L(2,11)$

Remark 3.5. *For $t > 2$ we refer the reader to [14]. The design 5-$(12,6,1)$ is famous Witt design, W_{12}, on which the Mathieu group M_{12} acts 5-transitively. For further information on W_{12} we refer the reader to [20, 21].*

3.6. **Transitive t-designs from $L(2,13)$.** The linear group $L(2,13)$ is the simple group of order 1092 and up to conjugation it has 16 subgroups. We classify t-designs, $t \geq 2$, $v < 78$ admitting transitive action of the group $L(2,13)$. These subgroups are listed in Table 14, and in Table 15 we give the information on the obtained designs.

TABLE 14. Subgroups of the group $L(2,13)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	1092	No	D_{12}	12	91	Yes
Z_2	2	546	No	A_4	12	91	Yes
Z_3	3	364	No	Z_{13}	13	84	No
E_4	4	273	No	D_{14}	14	78	Yes
S_3	6	182	No	D_{26}	26	42	No
S_3	6	182	No	$Z_{13} : Z_3$	39	28	No
Z_6	6	182	No	$Z_{13} : Z_6$	78	14	Yes
Z_7	7	156	No	$L(2,13)$	1092	1	No

TABLE 15. t-designs constructed from the group $L(2,13)$, $v < 78$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-$(14,4,3)$	273	1	$L(2,13) : Z_2$
3-$(14,4,6)$	546	1	$L(2,13) : Z_2$
3-$(14,5,15)$	546	1	$L(2,13) : Z_2$
3-$(14,6,5)$	91	3	$L(2,13) : Z_2$
3-$(14,6,30)$	546	1	$L(2,13) : Z_2$
3-$(14,6,60)$	1092	1	$L(2,13) : Z_2$
2-$(14,3,6)$	182	1	$L(2,13)$
2-$(14,4,6)$	91	1	$L(2,13)$
2-$(14,5,20)^*$	182	1	$L(2,13)$
2-$(14,5,60)^*$	546	1	$L(2,13)$
2-$(14,6,15)$	91	1	$L(2,13)$
2-$(14,6,90)^*$	546	1	$L(2,13)$
2-$(14,7,18)$	78	1	$L(2,13)$
2-$(14,7,42)^*$	182	1	$L(2,13)$
2-$(14,7,84)^*$	364	1	$L(2,13)$
2-$(14,7,126)^*$	546	2	$L(2,13)$

Remark 3.6. *For $t = 3$ we refer the reader to* [14].

3.7. **Transitive t-designs from $L(2,16)$.** The linear group $L(2,16)$ is the simple group of order 4080 and up to conjugation it has 21 subgroups, listed in Table 16. We classify t-designs, $t \geq 2$, $v < 51$ admitting transitive action of the group $L(2,16)$. The constructed designs are described in Table 17.

Additionally, we classify t-designs, $t \geq 2$, $v = 51$, $b \leq 2040$ admitting transitive action of the group $L(2,16)$ and the obtained designs are listed in Table 18.

Table 16. Subgroups of the group $L(2,16)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	4080	No	Z_{15}	15	272	No
Z_2	2	2040	No	E_{16}	16	255	No
Z_3	3	1360	No	Z_{17}	17	240	No
E_4	4	1020	No	D_{30}	30	136	Yes
E_4	4	1020	No	D_{34}	34	120	Yes
E_4	4	1020	No	$E_{16}:Z_3$	48	85	No
Z_5	5	816	No	A_5	60	68	Yes
S_3	6	680	No	$E_{16}:Z_5$	80	51	No
E_8	8	510	No	$E_{16}:Z_{15}$	240	17	Yes
D_{10}	10	408	No	$L(2,16)$	4080	1	No
A_4	12	340	No				

Table 17. t-designs constructed from the group $L(2,16)$, $v<51$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
4-$(17,7,6)$	408	1	$L(2,16):Z_4$
4-$(17,7,60)$	4080	1	$L(2,16):Z_4$
4-$(17,8,15)$	510	1	$L(2,16):Z_4$
4-$(17,8,120)$	4080	1	$L(2,16):Z_4$
3-$(17,3,1)$	680	1	S_{17}
3-$(17,4,2)$	340	1	$L(2,16):Z_4$
3-$(17,4,6)$	1020	1	$L(2,16):Z_2$
3-$(17,5,1)$	68	1	$L(2,16):Z_4$
3-$(17,5,15)$	1020	1	$L(2,16):Z_2$
3-$(17,5,60)$	4080	1	$L(2,16):Z_4$
3-$(17,6,20)$	680	1	$L(2,16):Z_2$
3-$(17,6,24)$	816	1	$L(2,16):Z_4$
3-$(17,6,60)$	2040	1	$L(2,16):Z_4$
		1	$L(2,16)$
3-$(17,7,70)$	1360	1	$L(2,16):Z_2$
3-$(17,7,105)$	2040	2	$L(2,16)$
3-$(17,8,56)$	680	2	$L(2,16):Z_2$
3-$(17,8,168)$	2040	1	$L(2,16):Z_4$
		1	$L(2,16)$
3-$(17,8,336)$	4080	1	$L(2,16):Z_2$

TABLE 18. t-designs constructed from the group $L(2,16)$, $v = 51, b \leq 2040$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(51,25,240)^*$	1020	2	$(Z_3 \times L(2,16)) : Z_4$
		2	$Z_3 \times (L(2,16) : Z_2)$
2-$(51,25,480)^*$	2040	117	$L(2,16)$
		4	$L(2,16) : Z_2$

Remark 3.7. *For designs with $t = 3,4$ we refer the reader to* [14].

3.8. **Transitive t-designs from $L(2,17)$.** The linear group $L(2,17)$ is the simple group of order 2448 and up to conjugation it has 22 subgroups. The subgroups are listed in Table 19, and the constructed designs are given in Table 20. We classify t-designs, $t \geq 2$, $v < 36$ admitting transitive action of the group $L(2,17)$ and the obtained designs are listed in Table 20. Additionally, we classify t-designs, $t \geq 2$, $v = 36$, $b \leq 1224$ admitting transitive action of the group $L(2,17)$ and the obtained designs are listed in Table 21.

TABLE 19. Subgroups of the group $L(2,17)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	2448	No	A_4	12	204	No
Z_2	2	1224	No	A_4	12	204	No
Z_3	3	816	No	D_{16}	16	153	Yes
E_4	4	612	No	Z_{17}	17	144	No
E_4	4	612	No	D_{18}	18	136	Yes
Z_4	4	612	No	S_4	24	102	Yes
S_3	6	408	No	S_4	24	102	Yes
D_8	8	306	No	D_{34}	34	72	No
D_8	8	306	No	$Z_{17} : Z_4$	68	36	No
Z_8	8	306	No	$Z_{17} : Z_8$	136	18	Yes
Z_9	9	272	No	$L(2,17)$	2448	1	No

TABLE 20. t-designs constructed from the group $L(2,17)$, $v < 36$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
5-$(18,8,16)$	2448	1	$L(2,17):Z_2$
3-$(18,4,6)$	1224	2	$L(2,17):Z_2$
3-$(18,5,15)$	1224	1	$L(2,17)$
3-$(18,6,10)$	408	1	$L(2,17):Z_2$
3-$(18,6,20)$	816	1	$L(2,17):Z_2$
3-$(18,6,30)$	1224	1	$L(2,17):Z_2$
3-$(18,6,60)$	2448	2	$L(2,17):Z_2$
3-$(18,8,42)$	612	1	$L(2,17):Z_2$
		2	$L(2,17)$
3-$(18,8,84)$	1224	2	$L(2,17):Z_2$
		2	$L(2,17)$
3-$(18,8,168)$	2448	1	$L(2,17):Z_2$
3-$(18,9,252)$	2448	1	$L(2,17):Z_2$
2-$(18,3,8)^*$	408	1	$L(2,17)$
2-$(18,4,12)^*$	306	2	$L(2,17)$
2-$(18,5,40)^*$	612	1	$L(2,17)$
2-$(18,5,80)^*$	1224	2	$L(2,17)$
2-$(18,6,10)$	102	1	$L(2,17)$
2-$(18,6,60)^*$	612	1	$L(2,17)$
2-$(18,6,120)^*$	1224	2	$L(2,17)$
2-$(18,6,240)^*$	2448	1	$L(2,17)$
2-$(18,7,168)^*$	1224	11	$L(2,17)$
2-$(18,7,336)^*$	2448	3	$L(2,17)$
2-$(18,8,28)^*$	153	1	$L(2,17)$
2-$(18,8,224)^*$	1224	2	$L(2,17)$
2-$(18,8,448)^*$	2448	4	$L(2,17)$
2-$(18,9,32)^*$	136	2	$L(2,17)$
2-$(18,9,72)^*$	306	1	$L(2,17)$
2-$(18,9,96)^*$	408	2	$L(2,17)$
2-$(18,9,144)^*$	612	1	$L(2,17)$
2-$(18,9,192)^*$	816	1	$L(2,17)$
2-$(18,9,288)^*$	1224	9	$L(2,17)$
2-$(18,9,576)^*$	2448	5	$L(2,17)$

Table 21. t-designs constructed from the group $L(2,17)$, $v=36, b \leq 1224$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
2-$(36,15,68)^*$	408	3	$L(2,17)$
2-$(36,15,136)^*$	816	7	$L(2,17)$
2-$(36,15,204)^*$	1224	57	$L(2,17)$

Remark 3.8. *For information on known* 3*-designs with 18 points the reader may consult* [14].

3.9. **Transitive t-designs from $L(2,19)$.** The linear group $L(2,19)$ is the simple group of order 3420 and up to conjugation it has 19 subgroups. We classify t-designs, $t \geq 2$, $v < 57$ admitting transitive action of the group $L(2,19)$. These subgroups are listed in Table 22, and the information on constructed designs are given in Table 23.

Table 22. Subgroups of the group $L(2,19)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	3420	No	A_4	12	285	No
Z_2	2	1710	No	D_{18}	18	190	Yes
Z_3	3	1140	No	Z_{19}	19	180	No
E_4	4	855	No	D_{20}	20	171	Yes
Z_5	5	684	No	$Z_{19}:Z_3$	57	60	No
S_3	6	570	No	A_5	60	57	Yes
Z_9	9	380	No	A_5	60	57	Yes
D_{10}	10	342	No	$Z_{19}:Z_9$	171	20	Yes
D_{10}	10	342	No	$L(2,19)$	3420	1	No
Z_{10}	10	342	No				

Table 23. t-designs constructed from the group $L(2,19)$, $v < 57$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-$(20,3,1)$	1140	1	S_{20}
3-$(20,4,1)$	285	1	$L(2,19)$
3-$(20,4,3)$	855	1	$L(2,19):Z_2$
3-$(20,4,6)$	1710	2	$L(2,19):Z_2$
3-$(20,5,6)$	684	1	$L(2,19):Z_2$
3-$(20,5,10)$	1140	1	$L(2,19):Z_2$
3-$(20,5,30)$	3420	2	$L(2,19):Z_2$
		1	$L(2,19)$
3-$(20,6,10)$	570	1	$L(2,19):Z_2$
		1	$L(2,19)$
3-$(20,6,20)$	1140	1	$L(2,19):Z_2$
3-$(20,6,30)$	1710	3	$L(2,19):Z_2$
		3	$L(2,19)$
3-$(20,6,60)$	3420	2	$L(2,19):Z_2$
		2	$L(2,19)$
3-$(20,7,35)$	1140	1	$L(2,19):Z_2$
		2	$L(2,19)$
3-$(20,7,105)$	3420	9	$L(2,19):Z_2$
		6	$L(2,19)$
3-$(20,8,14)$	285	1	$L(2,19):Z_2$
3-$(20,8,28)$	570	1	$L(2,19):Z_2$
		1	$L(2,19)$
3-$(20,8,42)$	855	1	$L(2,19):Z_2$
		1	$L(2,19)$
3-$(20,8,84)$	1710	3	$L(2,19):Z_2$
		6	$L(2,19)$
3-$(20,8,168)$	3420	7	$L(2,19):Z_2$
		11	$L(2,19)$
3-$(20,9,28)$	380	1	$L(2,19):Z_2$
3-$(20,9,84)$	1140	1	$L(2,19):Z_2$
		1	$L(2,19)$
3-$(20,9,252)$	3420	12	$L(2,19):Z_2$
		21	$L(2,19)$
3-$(20,10,36)$	342	1	$L(2,19):Z_2$
		1	$L(2,19)$
3-$(20,10,40)$	380	1	$L(2,19)$
3-$(20,10,120)$	1140	3	$L(2,19)$
3-$(20,10,180)$	1710	5	$L(2,19):Z_2$
		10	$L(2,19)$
3-$(20,10,360)$	3420	8	$L(2,19):Z_2$
		16	$L(2,19)$

Remark 3.9. *For information on known* 3*-designs with 20 points the reader may consult* [14].

3.10. **Transitive t-designs from $L(2,23)$.** The linear group $L(2,23)$ is the simple group of order 6072 and up to conjugation it has 23 subgroups, which are listed in Table 24. We classify t-designs, $t \geq 2$, $v < 253$ admitting transitive action of the group $L(2,23)$. The information on constructed designs are given in Tables 25 and 26.

TABLE 24. Subgroups of the group $L(2,23)$

Structure	Order	Index	Primitive	Structure	Order	Index	Primitive
I	1	6072	No	A_4	12	506	No
Z_2	2	3036	No	D_{12}	12	506	No
Z_3	3	2024	No	D_{12}	12	506	No
E_4	4	1518	No	A_4	12	506	No
E_4	4	1518	No	D_{22}	22	276	Yes
Z_4	4	1518	No	Z_{23}	23	264	No
Z_6	6	1518	No	S_4	24	253	Yes
S_3	6	1012	No	S_4	24	253	Yes
S_3	6	1012	No	D_{24}	24	253	Yes
D_8	8	759	No	$Z_{23} : Z_{11}$	253	24	Yes
Z_{11}	11	552	No	$L(2,23)$	6072	1	No
Z_{12}	12	506	No				

TABLE 25. 5-designs constructed from the group $L(2,23)$, $v < 253$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
5-$(24,7,3)$	6072	1	$L(2,23)$
5-$(24,8,1)$	759	1	M_{24}
5-$(24,9,6)$	2024	1	$L(2,23)$
5-$(24,10,36)$	6072	1	$L(2,23)$

Table 26. 3-designs constructed from the group $L(2,23)$, $v < 253$

Parameters of designs	# of blocks	# non-isomorphic	Full automorphism group
3-(24, 3, 1)	2024	1	S_{24}
3-(24, 4, 3)	1518	1	$L(2,23):Z_2$
		1	$L(2,23)$
3-(24, 4, 6)	3036	2	$L(2,23):Z_2$
3-(24, 5, 30)	6072	5	$L(2,23):Z_2$
		1	$L(2,23)$
3-(24, 6, 10)	1012	1	$L(2,23):Z_2$
		2	$L(2,23)$
3-(24, 6, 20)	2024	1	$L(2,23):Z_2$
3-(24, 6, 30)	3036	4	$L(2,23):Z_2$
		5	$L(2,23)$
3-(24, 6, 60)	6072	4	$L(2,23):Z_2$
		5	$L(2,23)$
3-(24, 7, 105)	6072	15	$L(2,23):Z_2$
		20	$L(2,23)$
3-(24, 8, 21)	759	1	$L(2,23):Z_2$
3-(24, 8, 42)	1518	1	$L(2,23):Z_2$
		1	$L(2,23)$
3-(24, 8, 84)	3036	6	$L(2,23):Z_2$
		14	$L(2,23)$
3-(24, 8, 168)	6072	15	$L(2,23):Z_2$
		44	$L(2,23)$
3-(24, 9, 84)	2024	3	$L(2,23):Z_2$
		1	$L(2,23)$
3-(24, 9, 252)	6072	27	$L(2,23):Z_2$
		93	$L(2,23)$
3-(24, 10, 180)	3036	10	$L(2,23):Z_2$
		28	$L(2,23)$
3-(24, 10, 360)	6072	24	$L(2,23):Z_2$
		132	$L(2,23)$
3-(24, 11, 45)	552	1	$L(2,23):Z_2$
3-(24, 11, 495)	6072	41	$L(2,23):Z_2$
		185	$L(2,23)$
3-(24, 12, 55)	506	1	$L(2,23):Z_2$
		2	$L(2,23)$
3-(24, 12, 60)	552	2	$L(2,23)$
3-(24, 12, 110)	1012	2	$L(2,23)$
3-(24, 12, 165)	1518	1	$L(2,23):Z_2$
		4	$L(2,23)$
3-(24, 12, 220)	2024	2	$L(2,23):Z_2$
		1	$L(2,23)$
3-(24, 12, 330)	3036	8	$L(2,23):Z_2$
		27	$L(2,23)$
3-(24, 12, 660)	6072	30	$L(2,23):Z_2$
		190	$L(2,23)$

Remark 3.10. *In [6] Cameron determined parameters for possible* 3*-designs obtained from the group* $L(2,q), q \equiv 3 (mod\ 4)$*, but there the* 3*-designs have not been enumerated. In [14, 15], the designs with parameters* 3-$(24,11,495)$ *and* 3-$(24,12,5m), m \in \{11,12,22,33,44,66,132\}$ *from Table 26 are mentioned as unknown. The design* 5-$(24,8,1)$ *is famous Witt design,* W_{24}*, on which the Mathieu group* M_{24} *acts* 5*-transitively. For further information on* W_{24} *we refer the reader to [20, 21].*

3.11. **SRGs from** $L(2,q), q \leq 23$**.** Using the method described in Theorem 2.1 and Corollary 2.2, we obtained regular graphs on which the linear groups $L(2,q), q \leq 23$ act transitively. Using the computer search we obtained strongly regular graphs. Finally, we determined the full automorphism groups of the constructed SRGs. In Table 27 we give the details about obtained graphs i.e. the linear group which acts transitively on the obtained graph, the parameters and the full automorphism group of the graph.

TABLE 27. SRGs from $L(2,q), q \leq 23$

Graph Γ	Parameters	$Aut(\Gamma)$
$\Gamma_1 = \Gamma(L(2,5))$	(10,3,0,1)	S_5
$\Gamma_2 = \Gamma(L(2,5))$	(15,6,1,3)	S_6
$\Gamma_3 = \Gamma(L(2,7))$	(21,10,3,6)	S_7
$\Gamma_4 = \Gamma(L(2,7))$	(28,12,6,4)	S_8
$\Gamma_5 = \Gamma(L(2,8))$	(36,14,7,4)	S_9
$\Gamma_6 = \Gamma(L(2,8))$	(63,30,13,15)	$L(2,8):Z_3$
$\Gamma_7 = \Gamma(L(2,8))$	(63,30,13,15)	$O(7,2)$
$\Gamma_8 = \Gamma(L(2,9))$	(15,6,1,3)	S_6
$\Gamma_9 = \Gamma(L(2,9))$	(36,10,4,2)	$(S_6 \times S_6):Z_2$
$\Gamma_{10} = \Gamma(L(2,9))$	(40,12,2,4)	$O(5,3):Z_2$
$\Gamma_{11} = \Gamma(L(2,9))$	(45,16,8,4)	S_{10}
$\Gamma_{12} = \Gamma(L(2,11))$	(55,18,9,4)	S_{11}
$\Gamma_{13} = \Gamma(L(2,11))$	(66,20,10,4)	S_{12}
$\Gamma_{14} = \Gamma(L(2,13))$	(91,24,12,4)	S_{14}
$\Gamma_{15} = \Gamma(L(2,16))$	(120,51,18,24)	$O(5,4):Z_2$
$\Gamma_{16} = \Gamma(L(2,16))$	(136,30,15,4)	S_{17}
$\Gamma_{17} = \Gamma(L(2,16))$	(136,60,24,28)	$O(5,4):Z_2$
$\Gamma_{18} = \Gamma(L(2,17))$	(136,63,30,28)	$L(2,17)$
$\Gamma_{19} = \Gamma(L(2,17))$	(153,32,16,4)	S_{18}
$\Gamma_{20} = \Gamma(L(2,19))$	(190,36,18,4)	S_{20}

The SRGs $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_8, \Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \Gamma_{14}, \Gamma_{16}, \Gamma_{19}, \Gamma_{20}$, are triangular graphs. The graph Γ_9 is the unique graph with this parameters. The graph Γ_{10} and Γ_7 are $O(5,3)$ and $O(7,2)$ graphs, respectively, they are obtained by taking the points on a nondegenerate quadric in PG(4,3) and PG(6,2), respectively, and defining the adjacency by orthogonality. The graphs Γ_{15} and Γ_{18} are polar graphs $NO^-(5,4)$ and $NO^-(8,2)$ respectively, while Γ_{17} is the complement of the polar graph $NO^+(5,4)$. For further information we refer the reader to [3, 4].

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