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# FAITHFUL REAL REPRESENTATIONS OF CYCLICALLY PINCHED ONE-RELATOR GROUPS

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Abstract. In [4, 5] using faithful complex representations of cyclically pinched and conjugacy pinched one-relator groups we proved that any limit group has a faithful representation in $PSL(2,\mathbb{C})$. Further this representation can be effectively constructed using the JSJ decomposition. In this note we show that any hyperbolic cyclically pinched one-relator group with maximal amalgamated subgroups in each factor has a 2-dimensional faithful real representation.

## 1. Introduction

In this note we prove directly that any hyperbolic cyclically pinched one-relator group with maximal amalgamated subgroups in each factor and a certain hyperbolic rank one extension of centralizers has a faithful representation in $PSL(2,\mathbb{R})$. This is part of a general investigation of real and complex representations of limit groups.

Recall that a group $G$ is *fully residually free* if given any finite set of elements $g_1,\dots,g_n \in G$ there exists a homomorphism $\phi : G \mapsto F$ with $F$ a free group such that $\phi(g_i) \neq 1$ for all $i = 1,\dots,n$. If $G$ is finitely generated it is called a *limit group*. Limit groups played a prominent role in the solution of the Tarski problems by Kharlampovich and Myasnikov ([7],[8],[9],[10],[11],[12]) and Sela ([16],[17],[18],[19],[20],[21]). In Sela's approach they arise as limit groups of homomorphisms into free group whence the name. An extensive structure theory of limit groups has been developed (see the above references). Perhaps the primary example of a limit group is an orientable surface group $S_g$. Originally this was proved to be residually free by G. Baumslag [3] using what is now called the *big*

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*powers argument*. As a result of these groups being commutative transitive, it follows from a result of B.Baumslag [2], that they are fully residually free. It is well known that orientable surface groups of genus $\geq 2$ have faithful representations in $PSL(2, \mathbb{R})$. In [15] it was proved that surface groups can be embedded in complex semisimple Lie groups.

In [4] it was proved directly using the JSJ decomposition of a limit group that any hyperbolic limit group can be faithfully represented in $PSL(2, \mathbb{C})$. Further this representation can be effectively constructed using knowledge of the JSJ decomposition. The results needed in [4] are that facts that hyperbolic cyclically pinched one-relator groups as well as certain conjugacy pinched one-relator groups have faithful representations in $PSL(2, \mathbb{C})$. In [5] the restriction to hyperbolic limit groups was removed using the embedding of any limit group in a nonstandard free group $F^*$. Our proof here depends on the free product with amalgamation decomposition of a cyclically pinched one-relator group but relies on the fact that the factors are free. Hence the general proof for limit groups as done in [4],[5] does not go through. It thus remains an open question whether all limit groups have faithful real 2-dimensional representations.

## 2. Faithful Real Representations of Hyperbolic Cyclically Pinched One-Relator Groups

The proof in [4] that a hyperbolic limit group has a faithful 2-dimensional complex representation depended upon the fact that such a group has a particular type of graph of groups decomposition - its JSJ decomposition. This decomposition was used effectively to prove that if $G$ is a hyperbolic limit group then $G$ has a faithful representation in $PSL(2, \mathbb{C})$. In an attempt to extend this to real representations we extended a result of Rosenberger [13] on faithful representations into $PSL(2, \mathbb{C})$ of cyclically pinched one-relator groups to real representations.

Recall that a **cyclically pinched one-relator group** is a one-relator group with a presentation of the following form

$$G = \langle a_1, \ldots, a_p, a_{p+1}, \ldots, a_n; U = V \rangle$$

where $1 \neq U = U(a_1, \ldots, a_p)$ is a cyclically reduced, non-primitive (not part of a free basis) word in the free group $F_1$ on $a_1, \ldots, a_p$ and $1 \neq V = V(a_{p+1}, \ldots, a_n)$ is a cyclically reduced, non-primitive word in the free group $F_2$ on $a_{p+1}, \ldots, a_n$.

Clearly such a group is the free product of the free groups on $a_1, \ldots, a_p$ and $a_{p+1}, \ldots, a_n$ respectively amalgamated over the cyclic subgroups generated by $U$ and $V$. If neither $U$ nor $V$ are proper powers in the respective free groups on $a_1, \ldots, a_p$ and $a_{p+1}, \ldots, a_n$ then the resulting group is hyperbolic (see [4]). Hyperbolic Surface groups are cyclically pinched one-relator groups and general hyperbolic cyclically pinched one-relator groups share many properties with surface groups. It is well-known that hyperbolic surface groups have faithful representations in $PSL(2, \mathbb{R})$. In [13] it was shown that a general hyperbolic cyclically pinched one-relator group has a faithful representation in $PSL(2, \mathbb{C})$. Here we extend this to faithful real representations.

**Theorem 2.1.** *Let* $G = \langle a_1, \ldots, a_n, b_1, \ldots, b_m; u = v \rangle$ *with* $n \geq 2, m \geq 2$ *and* $u = u(a_1, \ldots, a_n)$ *a nontrivial, not primitive and not a proper power element in the free group* $F_1 = \langle a_1, \ldots, a_n \rangle$ *and*

*$v = v(b_1, \ldots, b_m)$ a nontrivial, not primitive and not a proper power element in the free group $F_2 = \langle b_1, \ldots, b_m \rangle$.*

*Then there exists a faithful representation $\phi : G \to PSL(2, R)$*

*Proof.* We prove the existence of a faithful representation into $SL(2, \mathbb{R})$. Since a hyperbolic cyclically pinched one-relator group as in the statement of the theorem is centerless this faithful representation can be extended to a faithful representation into $PSL(2, \mathbb{R})$.

We first embed $F_1$ into a free group $H_1 = \langle a, b \rangle$ of rank 2 and $F_2$ into a free group $H_2 = \langle c, d \rangle$ also of rank 2. It follows that $U, V$ are both nontrivial and not primitive in $H_1, H_2$ respectively.

We now consider $H_1 = \langle a, b \rangle$. Choose $A, B \in SL(2, \mathbb{R})$ with $tr(A) = x > 2$ an algebraic number and $tr(B) = y > 2$ also an algebraic number. Then $tr(AB) = r$ and we will choose $r$ later in a suitable manner.

The map $a \mapsto A, b \mapsto B$ defines a homomorphism $\phi_1 : H_1 \to SL(2, \mathbb{R})$. Let $U = \phi_1(u)$ be the image of $u$. From [5],[6] and also from [14] page 97 we get the following: the trace $tr(U)$ is a nonconstant polynomial $f(r)$ in $r$ with coefficients in $\mathbb{Z}[x, y]$; and without loss of generality we may assume that the highest coefficient is positive. Moreover all the coefficients are algebraic numbers.

We now argue analogously for $H_2 = \langle c, d \rangle$. Choose $C, D \in SL(2, \mathbb{R})$ with $tr(C) = z > 2$ an algebraic number and $tr(D) = w > 2$ an algebraic number. We let $s = tr(CD)$ and, just as for $r$, we will later choose $s$ in a suitable manner.

As before the map $c \mapsto C, d \mapsto D$ defines a homomorphism $\phi_2 : H_2 \to SL(2, \mathbb{R})$. Let $V = \phi_2(v)$ be the image of $v$. As before $tr(V)$ is a nonconstant polynomial $g(s)$ in $s$ with coefficients in $\mathbb{Z}[z, w]$; and without loss of generality we may assume that the highest coefficient is positive. Moreover all the coefficients are algebraic numbers.

We make the following observations.

(1) $f(X) \to \infty$ if $X \to \infty$ and
$g(X) \to \infty$ if $X \to \infty$

(2) If we choose a sufficiently large transcendental number $t > 4$ then by the intermediate value theorem there exist $r \in \mathbb{R}$ and $s \in \mathbb{R}$ such that $f(r) = t = g(s)$.

The real numbers $r, s$ have to be transcendental because the polynomials $f(X)$ and $g(X)$ have algebraic coefficients (if $r$ were algebraic then $f(r)$ would also be algebraic).

After a suitable conjugation of $\phi_1(H_1)$ and $\phi_2(H_2)$ in $SL(2, \mathbb{R})$ we may assume that

$$U = \begin{pmatrix} t_1 & 0 \\ 0 & t_1^{-1} \end{pmatrix} = V, t = t_1 + t_1^{-1},$$

with $t_1$ a real transcendental (recall that $t > 4$).

We have the following facts.

(a) $\langle A, B \rangle$ is a free group of rank 2 because $r$ is transcendental.

(b) $\langle C, D \rangle$ is a free group of rank 2 because $s$ is transcendental.

Therefore $\phi_1$ and $\phi_2$ are monomorphisms and hence embeddings of the respective free groups into $SL(2,\mathbb{R})$. $F_1$ is a subgroup of $H_1 = \langle a,b\rangle$ and $F_2$ is a subgroup of $H_2 = \langle c,d\rangle$. Hence

$$\phi_1|F_1 : F_1 \to SL(2,\mathbb{R})$$

and

$$\phi_2|F_2 : F_2 \to SL(2,\mathbb{R})$$

are embeddings with

$$\phi_1|F_1(u) = U = V = \phi_2|F_2(v).$$

Recall that

$$U = \begin{pmatrix} t_1 & 0 \\ 0 & t_1^{-1} \end{pmatrix}.$$

Then the combination of $\phi_1|F_1$ and $\phi_2|F_2$ defines a homomorphism

$$\phi : G \to SL(2,\mathbb{R})$$

with

$$\phi|F_1 = \phi_1|F_1 \text{ and } \phi|F_2 = \phi_2|F_2.$$

Now, $u$ is not a proper power in $F_1$ and $v$ is not a proper power in $F_2$. Hence exactly as in [14] and [13] we get from the homomorphism $\phi : G \to SL(2,\mathbb{R})$ an injective homomorphism $\rho : G \to SL(2,\mathbb{R})$. To see this let

$$tr(G) = \{tr(\phi(g)); g \in G\}$$

and then choose a real transcendental number $\tau$ which is not algebraic over $K = \mathbb{Q}(tr(G))$. Now define

$$T = \begin{pmatrix} \tau & 0 \\ 0 & \tau^{-1} \end{pmatrix}.$$

Define a homomorphism $\rho : G \to SL(2,\mathbb{R})$ by $\rho(g) = T\phi(g)T^{-1}$ if $g \in F_1$ and $\rho(g) = \phi(g)$ if $g \in F_2$. Since $\rho(u) = \phi(u) = U = V$ this does indeed give a homomorphism.

For $g \in G$ we write $g_{ij}$ for the entry of $\rho(g)$ in the $ij$-th position. Then $\rho(g) = (g_{ij})$. We mention that $g_{12} \neq 0$ and $g_{21} \neq 0$ if $g \in F_1 \setminus \langle u\rangle$ or $g \in F_2 \setminus \langle v\rangle$ since $F_1, F_2$ are nonabelian and $u$ and $v$ are not proper powers in $F_1$ and $F_2$ respectively.

We now show that $\rho$ is injective. Every element $g \in G$ is conjugate either to an element of $F_1$ or $F_2$ or to an element of the form

$$g = x_1 y_1 \cdots x_k y_k$$

with $k \geq 1$ and $x_i \in F_1 \setminus \langle u\rangle$ and $y_i \in F_2 \setminus \langle v\rangle$ for $i = 1,\ldots,k$. To prove that $\rho$ is injective we may assume that $g = x_1 y_1 \cdots x_k y_k$ as above. We now show that $tr(\rho(g)$ is transcendental over $K$ which proves that $\rho$ is injective. For this we claim that $g_{11}$ is a Laurent polynomial of degree $2k \geq 2$ over $K$, $g_{12}$ is a Laurent polynomial of degree $\leq 2k$ over $K$ and $g_{12}$ and $g_{21}$ are Laurent polynomials of degree

$< 2k$ over $K$. We prove the claim by induction on $k$. If $k = 1$ then $g = xy$ with $x = x_1, y = y_1$. Recall that

$$\rho(x) = T\phi(x)T^{-1} = \begin{pmatrix} \tau & 0 \\ 0 & \tau^{-1} \end{pmatrix} \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \tau^{-1} & 0 \\ 0 & \tau \end{pmatrix}$$

with $\alpha_{12} \neq 0 \neq \alpha_{21}$ because $\phi$ is injective on $F_1$.

Then:

$$g_{11} = \alpha_{11}y_{11} + \alpha_{12}y_{12}\tau^2,$$

$$g_{12} = \alpha_{11}y_{12} + \alpha_{12}y_{22}\tau^2,$$

$$g_{21} = \alpha_{22}y_{11} + \alpha_{21}y_{11}\tau^{-2},$$

$$g_{22} = \alpha_{22}y_{11} + \alpha_{21}y_{21}\tau^{-2}.$$

Note that possibly $y_{22} = 0$. This proves the claim for $k = 1$.

Now let $k \geq 2$. We write $g = xy$ with $x = x_1y_1 \cdots x_{k-1}y_{k-1}$ and $y = x_ky_k$. By the inductive hypothesis and the case $k = 1$ already proven the claim holds for both $x$ and $y$. Multiplication of $\rho(x)$ with $\rho(y)$ now proves the overall claim. Hence in particular $tr(\rho(g))$ is transcendental over $K$ which proves that the homomorphism $\rho$ is injective. □

## 3. Faithful Real Representations of Hyperbolic Extensions of Centralizers

A **rank one extension of centralizers of** a group $G$ is a group

$$\Gamma = \langle G, t; rel(G), t^{-1}ut = u\rangle$$

where $\langle u\rangle$ is a maximal cyclic subgroup of $G$. It is known (see the references in [4]) that the class of limit groups consists of subgroups of the class of groups built up by iterated extensions of centralizers starting with free groups. The proof given in [4] that extensions of centralizers of groups admitting faithful complex representations also have faithful complex representations carries over to the real case.

**Theorem 3.1.** *Let $G$ be torsion-free, finitely generated and admit a faithful representation into $PSL(2,\mathbb{R})$. Suppose further that $G$ has cyclic centralizers. Let $\langle w\rangle$ be a maximal cyclic subgroup of $G$. Then the extension of centralizers*

$$\Gamma = \langle G, t; rel(G), twt^{-1} = w\rangle$$

*admits a faithful representation into $PSL(2,\mathbb{R})$.*

*Proof.* Let $G$ be finitely generated and admit a faithful representation into $PSL(2,\mathbb{R})$, let $\langle w\rangle$ be a maximal cyclic subgroup of $G$ and let

$$\Gamma = \langle G, t; rel(G), twt^{-1} = w\rangle.$$

We claim that if the centralizer of $w$ in $G$ is $\langle w\rangle$ then $\langle w\rangle$ is malnormal in $G$. Suppose that $gw^mg^{-1} = w^n$ in $G$ with $g \in G$. Using the faithful representation into $PSL(2,\mathbb{R})$ it would follow that

$g$ and $w$ have the same fixed points as mappings on $\mathbb{C} \cup \{\infty\}$ and hence $g, w$ commute. It follows that $g \in \langle w \rangle$ and hence $\langle w \rangle$ is malnormal in $G$.

Choose a representation $\phi$ into $PSL(2, \mathbb{R})$ so that

$$\phi(w) = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix}$$

or

$$\phi(w) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Suppose first that $\phi(w) = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix}$. Let $tr(G) = \{tr(g); g \in \phi(G)\}$. Choose a real transcendental number $\tau$ which is not algebraic over $K = \mathbb{Q}(tr(G))$. Extend $\phi$ to $\zeta$ so that $\phi = \zeta_{|G}$ and

$$\zeta(t) = \begin{pmatrix} \tau & 0 \\ 0 & \tau^{-1} \end{pmatrix}.$$

Then the map $\zeta : \Gamma \mapsto PSL(2, \mathbb{R})$ is injective.

Suppose next that $\phi(w) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Choose a transcendental number $\tau$ which is not algebraic over $K = \mathbb{Q}(tr(G))$. Extend $\phi$ to $\zeta$ so that $\phi = \zeta_{|G}$ and

$$\zeta(t) = \begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix}.$$

Then the map $\zeta : \Gamma \mapsto PSL(2, \mathbb{R})$ is injective.

That this map is injective follows in a similar manner as in the proof of Theorem 2.1. Using the normal form theorem for HNN groups we can consider a reduced word $g = g_1 t^{r_1} \cdots g_k t^{r_k}$ with $k > 0$, $r_i = 1$ or $r_i = -1$ for $i = 1, \ldots, k$ and $r_i = r_{i+1}$ if $k > 1$ and $g_{i+1}$ is in the associated subgroup $\langle w \rangle$ for $i = 1, \ldots, k-1$. As in the proof of Theorem 2.1, write $\zeta(g) = (g_{ij})$. Then $tr(\zeta(g))$ is transcendental over $K$ or $g_{21} \neq 0$ or $g_{12} \neq 0$

□

## 4. **Groups of F-Type**

In [6] Fine and Rosenberger introduced **groups of F-type** as a natural generalization of Fuchsian groups. In particular a group of F-type is a group of the form

$$G = \langle a_, \ldots, a_p, a_{p+1}, \ldots, a_n; a_i^{e_i} = 1, i = 1, \ldots, n, U = V \rangle$$

where $e_i = 0$ or $e_i > 1$ for $i = 1, \ldots, n$, $1 < p < n$, $U = U(a_1, \ldots, a_p)$ is of infinite order in the free product on $a_1, \ldots, a_p$ and $V = V(a_{p+1}, \ldots, a_n)$ is of infinite order in the free product on $a_{p+1}, \ldots, a_n$. This is a generalization of both a cyclically pinched one-relator group and the Poincare presentation of a finitely generated Fuchsian group. In [6] it was proved that such a group has a faithful representation in $PSL(2, \mathbb{C})$ and this was used to show that this class of groups shares many properties with Fuchsian groups (see [6]. The proof of Theorem 2.1 goes through without modification even if the generators have finite order. Hence we have the following which further enhances the similarity to Fuchsian groups. Recall that Fuchsian groups are defined as discrete subgroups of $PSL(2,\mathbb{R})$.

**Theorem 4.1.** *Let $G$ be a group of F-type. If the words $U$,$V$ are not proper powers in their respective free product of cyclics factors then $G$ has a faithful representation in $PSL(2,\mathbb{R})$.*

## 5. Obstruction to the Full Case of Limit Groups

The crux of the proof in [4] that all hyperbolic limit groups have faithful 2-dimensional complex representations depended upon considering extensions of centralizers of groups $B$ with the property that any subgroup of $B$ which has cyclic centralizers admits a faithful representation into $PSL(2,\mathbb{C})$. This in turn depended on the following:

**Lemma 5.1.** *Let $H_1, H_2$ be groups with cyclic centralizers and $u_1, u_2$ maximal cyclic elements in $H_1, H_2$ respectively. Suppose that $H_1, H_2$ admit faithful representations in $PSL(2,\mathbb{C})$ then the amalgamated free product $H = H_1 \underset{\{u_1=u_2\}}{\star} H_2$ also admits a faithful representation in $PSL(2,\mathbb{C})$.*

This lemma implies that free products of groups with cyclic centralizers admitting faithful representations in $PSL(2,\mathbb{C})$ also admit a faithful representation in $PSL(2,\mathbb{C})$.

**Corollary 5.2.** *Free products of groups with cyclic centralizers admitting faithful representations in $PSL(2,\mathbb{C})$ also admits a faithful representation in $PSL(2,\mathbb{C})$.*

If these could be proved in the real case the whole proof would go through. However our proof of Theorem 2.1 depends strongly on the fact that the factors are free groups. Hence it does not seem that this proof extends to Lemma 3.1 as it does in the complex case. Therefore the question of whether limit groups have faithful constructible 2-dimensional real representations remains open.

## References

[1] P. Ackermann, B. Fine and G. Rosenberger, On Surface Groups: Motivating Examples in Combinatorial Group Theory, *Groups St. Andrews 2005*, London Math. Soc. Lecture Notes Ser., Cambridge Univ. Press, Cambridge, **1** no. 339 (2007) 96–129.

[2] B. Baumslag, Residually free groups, *Proc. London Math. Soc. (3)*, **17** (1967) 402–418.

[3] G. Baumslag, On generalized free products, *Math. Z.*, **78** (1962) 423–438.

[4] B. Fine and G. Rosenberger, A note on Faithful Representations of Hyperbolic Limit Groups, *Groups Complex. Cryptol.*, **3** no. 2 (2011) 349–355.

[5] B. Fine and G. Rosenberger, Faithful Representations of Limit Groups II, *Groups Complex. Cryptol.*, to appear.

[6] B. Fine and G. Rosenberger, Generalizing Algebraic Properties of Fuchsian Groups, *Groups-St. Andrews 1989*, London Math. Soc., Lecture Note Ser., Cambridge Univ. Press, Cambridge, **1** no. 159 (1991) 124–147.

[7] O. Kharlamapovich and A. Myasnikov, Irreducible affine varieties over a free group: I. Irreducibility of quadratic equations and Nullstellensatz, *J. Algebra*, **200** (1998) 472–516.

[8] O. Kharlamapovich and A. Myasnikov, Affine varieties over a free group: II. Systems in triangular quasi-quadratic form and a description of residually free groups, *J. Algebra*, **200** (1998) 517–569.

[9] O. Kharlamapovich and A. Myasnikov, Implicit function theorem over free groups, *J. Algebra*, **290** (2005) 1–203.

[10] O. Kharlamapovich and A. Myasnikov, Effective JSJ decompositions, *Contemp. Math.*, **378** (2005) 87–212.

[11] O. Kharlamapovich and A. Myasnikov, Elementary theory of free nonabelian groups, *J. Algebra*, **302** (2006) 451–552.

[12] O. Kharlamapovich and A. Myasnikov, Hyperbolic groups and free constructions, *Trans. Amer. Math. Soc.*, **350** no. 2 (1998) 571–613.

[13] G. Rosenberger, Linear representations of cyclically pinched one-relator groups, *Siberian Math. J.*, **32** (1991) 203–206.

[14] P. Shalen, Linear representations of certain amalgamated products, *J. Pure Appl. Algebra*, **15** (1979) 187-197.

[15] A. S. Rapinchuk, V. V. Benyash-Krivetz and V. I. Chernousov, Representation varieties of the fundamental groups of compact orientable surfaces, *Israel J. Math.*, **93** (1996) 29–71.

[16] Z. Sela, The isomorphism problem for hyperbolic groups I., *Ann. of Math. (2)*, **141** no. 2 (1995) 217–283.

[17] Z. Sela, Diophantine geometry over groups I: Makanin-Razborov diagrams, *Publ. Math. Inst. Hautes Etudes Sci.*, no. 93 (2001) 31–105.

[18] Z. Sela, Diophantine geometry over groups II: Completions, closures and formal solutions, *Israel J. Math.*, **134** (2003) 173–254.

[19] Z. Sela, Diophantine geometry over groups III: Rigid and solid solutions, *Israel J. Math.*, **147** (2005) 1–73.

[20] Z. Sela, Diophantine geometry over groups IV: An iterative procedure for validation of a sentence, *Israel J. Math.*, **143** (2004) 1–130.

[21] Z. Sela, Diophantine geometry over groups V: Quantifier elimination, *Israel J. Math.*, **150** (2005) 1–197.

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