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A CHARACTERIZATION OF $L_2(81)$ BY NSE

LEILA MOUSAVI* AND BIJAN TAERI

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Abstract. Let $\pi_e(G)$ be the set of element orders of a finite group G. Let $\mathrm{nse}(G) = \{m_n \mid n \in \pi_e(G)\}$, where m_n be the number of elements of order n in G. In this paper, we prove that if $\mathrm{nse}(G) = \mathrm{nse}(L_2(81))$, then $G \cong L_2(81)$.

1. **Introduction**

For a finite group G and a positive integer t, let $M_t(G)$ be the set of all elements satisfying the equation $x^t = 1$, that is $M_t(G) = \{g \in G \mid g^t = 1\}$. The groups G_1 and G_2 are called of the same order type if and only if $|M_t(G_1)| = |M_t(G_2)|$, $t = 1, 2, \ldots$. In 1987, J. G. Thompson posed a question as follows:

Thompson's Problem. Suppose that G_1 and G_2 are of the same order type. If G_1 is solvable, is it true that G_2 is also necessarily solvable?

Let $\mathrm{nse}(G) = \{m_n \mid n \in \pi_e(G)\}$, where m_n is the number of elements of order n in G and $\pi_e(G)$ is the set of element orders of G. It is well known that if G_1 and G_2 are of the same order type, then $|G_1| = |G_2|$ and $\mathrm{nse}(G_1) = \mathrm{nse}(G_2)$. So it is natural to investigate Thompson's problem by $|G|$ and $\mathrm{nse}(G)$. The following example of Thompson shows that there are finite groups which are not characterizable by $\mathrm{nse}(G)$ and $|G|$. For the groups $G_1 = (C_2 \times C_2 \times C_2 \times C_2) \rtimes A_7$ and $G_2 = L_3(4) \rtimes C_2$ (which are maximal subgroups of M_{23}, the Mathieu group of degree 23) we have $\mathrm{nse}(G_1) = \mathrm{nse}(G_2)$ and $|G_1| = |G_2|$ but $G_1 \ncong G_2$.

In [8] Khatami, Khosravi, Akhlaghi posed the following question:

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∗Corresponding author.

Problem. Let G be a group such that $\mathrm{nse}(G) = \mathrm{nse}(L_2(q))$, where q is a prime power. Is G isomorphic to $L_2(q)$?

Affirmative answer is given for a few groups. In [1, 2, 8, 11] it is proved that $A_4 \cong L_2(3), A_5 \cong L_2(4) \cong L_2(5), A_6 \cong L_2(9)$, $L_2(7)$, $L_2(8)$, $L_2(11)$, $L_2(13)$, $L_2(25)$ and $L_2(27)$ are uniquely determined by nse. Recently, Shao and Jiang [9] proved that $L_2(p)$ are characterizable only by $\mathrm{nse}(G)$ for prime number $p \geq 5$. We continue the investigation of this problem and show that $L_2(81)$ is characterizable by nse.

Main Theorem. Let G be a group such that $\mathrm{nse}(G) = \mathrm{nse}(L_2(81))$. Then $G \cong L_2(81)$.

Our notations are standard and the readers may refer to [3].

2. Preliminary Results

In this section we present some useful lemmas which will be used in the proof of main theorem.

Lemma 2.1. [4] *Let G be a finite group and t be a positive integer dividing $|G|$. If $M_t(G) = \{g \in G \mid g^t = 1\}$, then $t \mid |M_t(G)|$.*

Lemma 2.2. [5] *Let G be a finite solvable group and $|G| = mn$, where $m = p_1^{\alpha_1} \dots p_r^{\alpha_r}$, $(m, n) = 1$. Let $\pi = \{p_1, \dots, p_r\}$ and h_m be the number of π-Hall subgroups of G. Then $h_m = q_1^{\beta_1} \cdots q_s^{\beta_s}$ satisfies the following conditions for all $i \in \{1, 2, \dots, s\}$:*

(1) $q_i^{\beta_i} \equiv 1 \pmod{p_j}$, for some p_j;

(2) The order of some chief factor of G is divided by $q_i^{\beta_i}$.

Recall that a finite group G is called a simple K_n-group, if G is a simple group with $|\pi(G)| = n$, where $\pi(G)$ is the set of all primes dividing $|G|$.

Lemma 2.3. [6] *If G is a simple K_3-group, then $G \cong A_5$, A_6, $L_2(7)$, $L_2(8)$, $L_2(17)$, $L_3(3)$, $U_3(3)$ or $U_4(2)$.*

Lemma 2.4. [12] *Let G be a simple K_4-group. Then G is isomorphic to one of the following groups:*

(1) A_7, A_8, A_9, A_{10}.

(2) M_{11}, M_{12}, J_2.

(3) (a) $L_2(r)$, where r is a prime and satisfies $r^2 - 1 = 2^a \cdot 3^b \cdot v^c$ with $a \geq 1, b \geq 1, c \geq 1$ and $v > 3$ is a prime;

(b) $L_2(2^m)$, where m satisfies $2^m - 1 = u$, $2^m + 1 = 3t^b$, with $m \geq 2$, u, t are primes, $t > 3, b \geq 1$;

(c) $L_2(3^m)$, where m satisfies $3^m + 1 = 4t$, $3^m - 1 = 2u^c$ or $3^m + 1 = 4t^b$, $3^m - 1 = 2u$, with $m \geq 2$, u, t odd primes, $b \geq 1$, $c \geq 1$;

(d) $L_2(16)$, $L_2(25)$, $L_2(49)$, $L_2(81)$, $L_3(4)$, $L_3(5)$, $L_3(7)$, $L_3(8)$, $L_3(17)$, $L_4(3)$, $S_4(4)$, $S_4(5)$, $S_4(7)$, $S_4(9)$, $S_6(2)$, $O_8^+(2)$, $G_2(3)$, $U_3(4)$, $U_3(5)$, $U_3(7)$, $U_3(8)$, $U_3(9)$, $U_4(3)$, $U_5(2)$, $S_z(8)$, $S_z(32)$, ${}^3D_4(2)$, ${}^2F_4(2)'$.

Lemma 2.5. [11] *Let G be a group containing more than two elements. If $s = \sup\{m_n \mid n \in \pi_e(G)\}$ is finite, then G is finite and $|G| \leq s(s^2-1)$.*

Let $r \in \pi(G)$. We denote by P_r, $\mathrm{Syl}_r(G)$ and $n_r(G)$, a Sylow r-subgroup, the set of Sylow r-subgroups of G and $|\mathrm{Syl}_r(G)|$, respectively.

Lemma 2.6. [10] *Let G be a finite group and let $P \in \mathrm{Syl}_p(G)$, where $p \in \pi(G)$. Let G have a normal series $K \trianglelefteq L \trianglelefteq G$. If $P \leq L$ and $p \nmid |K|$, then the following hold:*
(1) $N_{G/K}(PK/K) = N_G(P)K/K$;
(2) $|G : N_G(P)| = |L : N_L(P)|$, *that is,* $n_p(G) = n_p(L)$;
(3) $|L/K : N_{L/K}(PK/K)|t = |G : N_G(P)| = |L : N_L(P)|$, *that is,* $n_p(L/K)t = n_p(G) = n_p(L)$ *for some positive integer t, and* $|N_K(P)|t = |K|$.

Shao, Shi and Jiang proved that all simple K_4-groups G can be uniquely determined by $|G|$ and $\mathrm{nse}(G)$:

Theorem 2.7. [10] *Let G be a group and M a simple K_4-group. Then $G \cong M$ if and only if the following hold:*
(1) $|G| = |M|$;
(2) $\mathrm{nse}(G) = \mathrm{nse}(M)$.

From now on, we assume that G is a group such that $\mathrm{nse}(G) = \mathrm{nse}(L_2(81))$. By Lemma 2.5, we can assume that G is finite. We know that $m_n = k\phi(n)$, where k is the number of cyclic subgroups of order n in G and $\phi(n)$ the Euler totient function of n. Also we note that if $n > 2$, then $\phi(n)$ is even. If $n \in \pi_e(G)$, then by Lemma 2.1 and the discussion above we have:

$$(*) \qquad \begin{cases} \phi(n) \mid m_n \\ n \mid \Sigma_{d|n} m_d \end{cases}$$

In the proof of the main theorem we often apply (*) and the comment above. Using Theorems 8.2-8.5 of [7] we obtain that $\mathrm{nse}(L_2(81)) = \{1,\ A,\ B,\ C,\ D,\ E,\ F,\ H\}$, where

$$A := 3321,\ B := 6642,\ C := 6560,\ D := 13284,\ E := 53136,\ F := 26568,\ H := 129600.$$

3. Proof of the Main Theorem

Let G be a group such that $\mathrm{nse}(G) = \mathrm{nse}(L_2(81)) = \{1,\ A,\ B,\ C,\ D,\ E,\ F,\ H\}$. First we prove that $\pi(G) \subseteq \{2,\ 3,\ 5,\ 41\}$. Since $3321 \in \mathrm{nse}(G)$, it follows that $2 \in \pi(G)$ and $m_2 = A$. If $2 \neq p \in \pi(G)$ then by (*), $p \mid (1 + m_p)$ and $(p-1) \mid m_p$. This implies that $p \in \{2,\ 3\ ,5,\ 7,\ 41,\ 109,\ 163\}$. Therefore $\pi(G) \subseteq \{2,\ 3,\ 5,\ 7,\ 41,\ 109,\ 163\}$. If $3,\ 5,\ 7,\ 41,\ 109$ and $163 \in \pi(G)$ then by (*) we have $m_3 = C$, $m_5 = D$, $m_{41} = H$, $m_{109} = H$ and $m_{163} = F$. Also $m_7 \in \{B,\ E\}$. Now we show that $7,\ 109$ and $163 \notin \pi(G)$. Suppose that $2 \cdot 109 \in \pi_e(G)$, then by (*) $m_{2\cdot109} \in \{D,\ E,\ F,\ H\}$. But $2 \cdot 109 \mid (1 + m_2 + m_{109} + m_{2\cdot109}) = (1 + A + H + T)$, where $T \in \{D,\ E,\ F,\ H\}$, which is a contradiction. So $2\cdot 109 \notin \pi_e(G)$. Thus P_{109} acts fixed point freely on $\Omega_2 = \{g \in G \mid o(g) = 2\}$, the set of elements of order 2. Then $|P_{109}| \mid m_2$, a contradiction. Similarly, we can prove that $7,\ 163 \notin \pi(G)$. Thus $\pi(G) \subseteq \{2,\ 3,\ 5,\ 41\}$.

If $2^a \in \pi_e(G)$ then $\phi(2^a) = 2^{a-1} \mid m_{2^a}$ and so $0 \leq a \leq 7$.

If $3^b \in \pi_e(G)$ then $\phi(3^b) = 3^{b-1}\cdot 2 \mid m_{3^b}$ and so $0 \leq b \leq 5$.

If $5^c \in \pi_e(G)$ then $\phi(5^c) = 5^{c-1}\cdot 4 \mid m_{5^c}$ and so $0 \leq c \leq 3$.

If $41^d \in \pi_e(G)$ then $\phi(41^d) = 41^{d-1}\cdot 40 \mid m_{41^d}$ and so $0 \leq d \leq 2$.

Since $\exp(P_2) = 2,\ 4,\ 8,\ 16,\ 32,\ 64$, or 128, Lemma 2.1 implies that $|P_2| \mid 2^{10}$. If $3 \in \pi_e(G)$, then $\exp(P_3) = 3,\ 9,\ 27,\ 81,\ 243$, and by Lemma 2.1, $|P_3| \mid 3^8$. If $5 \in \pi_e(G)$ then $\exp(P_5) = 5,\ 25$, or 125. If $\exp(P_5) = 25$ or 125, then $m_{25} \in \{C,\ H\}$ and $m_{125} = H$, by (*). So Lemma 2.1 implies that $|P_5| = 5$, which is a contradiction. Therefore $\exp(P_5) = 5$. Since by Lemma 2.1 $|P_5| \mid (1 + m_5)$ we have $|P_5| = 5$. If $\exp(P_{41}) = 41^2$, then $m_{41^2} = C$. Hence $|P_{41}| = 41^2$, by Lemma 2.1. Therefore $n_{41} = \frac{m_{41^2}}{\phi(41^2)} = 4$. Thus $m_{41} + m_{41^2} \leq 4\cdot 41^2 = 6724$, which is a contradiction. So $\exp(P_{41}) = 41$ and thus $|P_{41}| = 41$.

Now suppose, if possible that, $2\cdot 41 \in \pi_e(G)$. Then $\phi(2\cdot 41) \mid m_{2\cdot 41}$ and so $m_{2\cdot 41} \in \{C, H\}$. Also $2\cdot 41 \mid (1 + m_2 + m_{41} + m_{2\cdot 41})$. Hence $2\cdot 41 \mid (1 + A + H + T)$, where $T \in \{C,\ H\}$ and so $m_{2\cdot 41} = C$. Recall that if P and Q are Sylow subgroups of G, then P and Q are conjugate and so $C_G(P)$ and $C_G(Q)$ are conjugate. Now since $m_{2\cdot 41} = \phi(2\cdot 41)\cdot t$, where t is the number of cyclic subgroups of order $2\cdot 41$, we conclude that $m_{2\cdot 41} = \phi(2\cdot 41)\cdot n_{41}(G)\cdot k$, where k is the number of cyclic subgroups of order two in $C_G(P_{41})$. Hence $m_{2\cdot 41} = m_{41}\cdot k$, which is a contradiction. Similarly, we can prove that G has not an element of order $5\cdot 41$. Also by (*) we can see that $3\cdot 41 \notin \pi_e(G)$.

Now we claim that $\pi(G) = \{2,\ 3,\ 5,\ 41\}$. Since $\pi(G) \subseteq \{2,\ 3,\ 5,\ 41\}$ to prove the claim we must consider the following cases:

Case a. $\pi(G) = \{2\}$. In this case we have

$$|G| = 239112 + Bk_1 + Ck_2 + Dk_3 + Ek_4 + Fk_5 + Hk_6 = 2^m,$$

where $k_1, \ldots, k_6$ and m are non-negative integers such that $0 \leq \Sigma_{i=1}^{6} k_i \leq 0$ and $1 \leq m \leq 10$. So $k_i = 0$, for $i = 1, \ldots, 6$. Thus $239112 \leq |G| \leq 2^{10}$, which is a contradiction.

Case b. $\pi(G) = \{2,\ 3\}$. Then $|G| = 239112 + Bk_1 + Ck_2 + Dk_3 + Ek_4 + Fk_5 + Hk_6 = 2^m.3^n$. As $\pi_e(G) \subseteq \{1, 2, 4, 8, 16, 32, 64, 128\} \cup \{3, 9, 27, 81, 243\} \cup \{2^i\cdot 3^j \mid 1 \leq i \leq 7 \text{ and } 1 \leq j \leq 5\}$, $0 \leq \Sigma_{i=1}^{6} k_i \leq 40$. Using a simple GAP program [13] (the function $f(L, n, p, x0)$, for $p = 2$, $x0 = 3322$, $1 \leq n \leq 6$; and $p = 3$, $x0 = 6561$, $1 \leq n \leq 4$, in Appendix), we see that $m \leq 10$ and $5 \leq n \leq 8$. Since $81 \mid |G|$ we have $81 \mid k_2$ and so $k_2 = 0$. Using the function $h(2^m\cdot 3^n)$, for $1 \leq m \leq 10$, $1 \leq n \leq 8$, in Appendix, we see that the equation has no solution unless $(m, n) \in \{(9, 8), (10, 7)\}$. But in these cases, the functions "f" that was mentioned above, show that there exists $n' \in \pi_e(G)$ and $n' \neq 3$ such that $m_{n'} = C$, contradicting $k_2 = 0$.

Case c. Let $5 \in \pi(G)$ or $41 \in \pi(G)$. Since $n_5 = \dfrac{m_5}{\phi(5)} = 3^4{\cdot}41 \mid |G|$ and $n_{41} = \dfrac{m_{41}}{\phi(41)} = 2^3{\cdot}3^4{\cdot}5 \mid |G|$, we have $\pi(G) = \{2,\ 3,\ 5,\ 41\}$.

Therefore the claim is proved, that is $\pi(G) = \{2,\ 3,\ 5,\ 41\}$. Since $2 \cdot 41 \notin \pi_e(G)$, the group P_2 acts on the set of elements of order 41 fixed point freely. Therefore $|P_2| \mid m_{41}$ and hence $|P_2| \mid 2^6$. Similarly, we can prove that $|P_3| \mid m_{41}$ and so $|P_3| \mid 3^4$. Hence $|G| = 2^m \cdot 3^4 \cdot 5 \cdot 41$, where $3 \leq m \leq 6$, as $n_5 \mid |G|$ and $n_{41} \mid |G|$. If $\exp(P_2) = 2$, then $|P_2| \mid (1 + m_2)$ and so $|P_2| = 2$, a contradiction. Similarly, $\exp(P_2) \neq 4$. Therefore $\exp(P_2) = 8,\ 16,\ 32$ or 64. We consider the following cases:

Case 1. $|G| = 2^3 \cdot 3^4 \cdot 5 \cdot 41 = 132840$. This case cannot happen, since $|G| \geq 239112$.

Case 2. $|G| \in \{2^4 \cdot 3^4 \cdot 5 \cdot 41,\ 2^5 \cdot 3^4 \cdot 5 \cdot 41,\ 2^6 \cdot 3^4 \cdot 5 \cdot 41\}$. If G is solvable, since $n_{41} = 2^3 \cdot 3^4 \cdot 5$, Lemma 2.2 shows that $8 \equiv 1 \pmod{41}$, which is a contradiction. Hence G is not solvable. Now let K be a maximal solvable normal subgroup of G and L/K be a minimal normal subgroup of G/K. Note that L/K is non-solvable, by maximality of K. So L/K is a simple group or a direct product of isomorphic simple groups. Let $L/K \cong S_1 \times \cdots \times S_r$, where S_1 is a non-abelian simple group and $S_1 \cong \cdots \cong S_r$. Now S_1 is a simple K_i-group, $i = 3,\ 4$, as $\pi(L/K) \subseteq \{2, 3, 5, 41\}$. Since $5^2 \nmid |G|$ and $41^2 \nmid |G|$ we conclude that $r = 1$. Therefore L/K is a simple K_i-group, where $i \in \{3,\ 4\}$.

Subcase (I). Let L/K be a simple K_3-group. Using Lemma 2.3, we can see that L/K is isomorphic to $A_5,\ A_6$ or $U_4(2)$. Let $L/K \cong A_5$. Then $|L/K| = 60$ and so $5 \nmid |K|$. By Lemma 2.6, there exists a positive integer t with $5 \nmid t$ such that $n_5(L/K)t = n_5(G)$. Therefore $6t = 3^4 \cdot 41$, which is a contradiction. Similarly, $L/K \ncong A_6,\ U_4(2)$.

Subcase (II). Let L/K be a simple K_4-group. Using Lemma 2.4, we can see that L/K is isomorphic to $L_2(81)$. Put $A/K := C_{G/K}(L/K)$. Then $A/K \cap L/K = 1$. Since $A/K, L/K \trianglelefteq G/K$ we have $A/K \times L/K \leq G/K$. Therefore $L/K \leq G/A \leq$ Aut(L/K). Since $\mathrm{Out}(L_2(81)) \cong \mathbb{Z}_2 \times \mathbb{Z}_4$ it follows that $|G/A| \in \{|L_2(81)|,\ 2|L_2(81)|, 4|L_2(81)|\}$. Hence $|A| \in \{1,\ 2,\ 4\}$. Since K is a maximal normal solvable subgroup of G, we have $A = K$. Therefore $L_2(81) \leq G/K \leq \mathrm{Aut}(L_2(81)) = P\Gamma L_2(81) = PGL_2(81) : \mathbb{Z}_4$.

If $K \neq 1$, then as $2 \cdot 41 \notin \pi_e(G)$, the action of P_{41} on K is fixed point free and so $|P_{41}| \mid (|K| - 1)$, which is a contradiction. Hence $K = 1$ and so G is a subgroup of $\mathrm{Aut}(L_2(81))$. Now suppose, if possible that $|G| = 2^5 \cdot 3^4 \cdot 5 \cdot 41$. It is well known that every element of $\mathrm{Out}(L_2(81))$ is a product of a field automorphism and a diagonal automorphism. If G contains a diagonal automorphism, then $G \cong PGL_2(81)$ and if G contains a field automorphism, then $G \cong L_2(81) : \mathbb{Z}_2$. In any case we have $m_2(L_2(81)) < m_2(G)$ and consequently $\mathrm{nse}(G) \neq \mathrm{nse}(L_2(81))$. If G contains a diagonal-field automorphism, then G is non-split extension of $L_2(81)$ by $\mathbb{Z}_2$ and by an easy GAP program [13] we see that $4 \in \pi_e(G)$ and $m_4(G) = 33210 \notin \mathrm{nse}(G)$. Now suppose, if possible that $|G| = 2^6 \cdot 3^4 \cdot 5 \cdot 41$. So $|\mathrm{Aut}(L_2(81)) : G| = 2$ and G is a normal subgroup of $\mathrm{Aut}(L_2(81))$. In this case we have $\mathrm{nse}(G) \neq \mathrm{nse}(L_2(81))$. This final contradiction shows that $|G| = 2^4 \cdot 3^4 \cdot 5 \cdot 41$ and hence by Theorem 2.7 we conclude that $G \cong L_2(81)$.

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Appendix

Let $L := \{6642, 6560, 13284, 53136, 26568, 129600\}$. The following function $f(L, n, p, x0)$ determines the maximum number m such that p^m divides $x_0 + x_1 + \cdots + x_n$, where $x_i \in L$, $1 \le i \le n$; note that some x_i's may be equal. We need the values of $f(L, n, 2, 3322)$, for $n = 1, 2, \ldots, 6$; and $f(L, n, 3, 6561)$, for $n = 1, 2, \ldots, 4$, in the proof of main theorem.

```
f:=function(L,n,p,x0) # 1<= n<=6 is a positive integer and L is a list of integers
 local a, x, max, t, m;
 max:=0;  a:=Tuples(L,n);
 for x in a do   Sort(x);   od;
 a:=Set(a);
 for x in a do
     t:=x0+Sum(x);
     m:=Number(Factors(t),x->x=p);
     if m > max then max:=m; Print(x);
     fi;
 od;
return max;
end;;
#############################################
```

The following function h returns the solution of the equation $|G| = 239112 + Bk_1 + Dk_3 + Ek_4 + Fk_5 + Hk_6 = 2^m.3^n$, where $0 \le \Sigma_{i=1}^{6} k_i \le 40$. We need $h(2^m \cdot 3^n)$, where $1 \le m \le 10$ and $1 \le n \le 8$, in the proof of main theorem, Case b.

```
h:=function(s)
   local a, x, L2, m, k6, res;
   res:=[];   L2:=[6642,13284,53136,26568];  m:=s-239112;
   if m<0 then return [];   fi;
   a:=Tuples([0..40],4);   a:=Filtered(a, x->Sum(x)<= 40);
   for x in a do
      k6:=(m-L2*x)/129600;
      if (not IsInt(k6)) or k6<0 then continue;   fi;
      if Sum(x)+k6<=40 then Add(res,Flat([x,k6]));   fi;
  od;
return res;
end;;
```

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Leila Mousavi

Department of Mathematical Sciences, Isfahan University of Technology, 84156-83111, Isfahan, Iran

Email: `l.mousavi@math.iut.ac.ir`

Bijan Taeri

Department of Mathematical Sciences, Isfahan University of Technology, 84156-83111, Isfahan, Iran

Email: `b.taeri@cc.iut.ac.ir`