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A NOTE ON THE POWER GRAPH OF A FINITE GROUP

Z. MEHRANIAN, A. GHOLAMI AND A. R. ASHRAFI*

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Abstract. Suppose Γ is a graph with $V(\Gamma) = \{1, 2, \ldots, p\}$ and $\mathcal{F} = \{\Gamma_1, \ldots, \Gamma_p\}$ is a family of graphs such that $n_j = |V(\Gamma_j)|$, $1 \leq j \leq p$. Define $\Lambda = \Gamma[\Gamma_1, \ldots, \Gamma_p]$ to be a graph with vertex set $V(\Lambda) = \bigcup_{j=1}^{p} V(\Gamma_j)$ and edge set $E(\Lambda) = \left(\bigcup_{j=1}^{p} E(\Gamma_j)\right) \cup \left(\bigcup_{ij \in E(\Gamma)} \{uv; u \in V(\Gamma_i), v \in V(\Gamma_j)\}\right)$. The graph Λ is called the Γ-join of $\mathcal{F}$. The power graph $\mathcal{P}(G)$ of a group G is the graph which has the group elements as vertex set and two elements are adjacent if one is a power of the other. The aim of this paper is to prove that $\mathcal{P}(\mathbb{Z}_n) = K_{\phi(n)+1} + \Delta_n[K_{\phi(d_1)}, K_{\phi(d_2)}, \ldots, K_{\phi(d_p)}]$, where Δ_n is a graph with vertex and edge sets $V(\Delta_n) = \{d_i \mid 1, n \neq d_i | n, 1 \leq i \leq p\}$ and $E(\Delta_n) = \{d_i d_j \mid d_i | d_j, 1 \leq i < j \leq p\}$, respectively. As a consequence it is proved that $Aut(\mathcal{P}(\mathbb{Z}_n)) \cong S_{\phi(n)+1} \times \prod_{1,n \neq d|n} S_{\phi(d)}$. This proves a recent conjecture by Doostabadi et al. [A. Doostabadi, A. Erfanian and A. Jafarzadeh, *Some results on the power graph of groups*, The Extended Abstracts of the 44th Annual Iranian Mathematics Conference, 27–30 August 2013, Ferdowsi University of Mashhad, Iran]. Finally, we apply our results to obtain complete descriptions of the power graphs of some finite groups.

1. Introduction

All groups and graphs in this paper are assumed to be finite. Suppose G is a finite group. The **power graph** $\mathcal{P}(G)$ is a graph in which $V(\mathcal{P}(G)) = G$ and two distinct elements x and y are adjacent if and only if one of them is a power of the other. The investigation of graphs related to groups as well as other algebraic structures is very important, because such graphs have valuable applications (see [7]) and are related to automata theory (see [8, 9]). These graphs were introduced in [6], see also [10, 11, 12].

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∗Corresponding author.

We refer to [13] for a survey of all recent results on the power graphs. Let us include only a brief overview of some relevant facts here. Chakrabarty et al. [4] proved that for a finite group G, $\mathcal{P}(G)$ is complete if and only if G is a cyclic group of order 1 or p^m, for some prime number p and positive integer m. They also obtained a formula for the number of edges in a finite power graph. Cameron and Ghosh [1] proved that non-isomorphic finite groups may have isomorphic power graphs, but that finite abelian groups with isomorphic power graphs must be isomorphic. They also show that the only finite group whose automorphism group is the same as its power graph is the Klein group of order 4. In this paper, the authors conjectured that two finite groups with isomorphic power graphs have the same number of elements of each order. Cameron [2] proved that in a finite group, the undirected power graph determines the directed power graph up to isomorphism. As a consequence, he responded affirmatively to the main conjecture of [1]. Pourgholi et al. [17], presented counterexamples for a conjecture maid by Chakrabarty et al. [3] regarding the values of n for which $P(U_n)$ is Hamiltonian. They also proved some results about characterization of simple groups by power graphs. Moghaddamfar et al. [15] considered the proper power graph $\mathcal{P}^\star(G)$. This graph can be constructed from $\mathcal{P}(G)$ by deleting the identity element of G. They provided necessary and sufficient conditions for a proper power graph $\mathcal{P}^\star(G)$ to be a strongly regular graph, a bipartite graph or a planar graph. They also obtained some infinite families of finite groups G for which the power graph $\mathcal{P}^\star(G)$ contains some cut-edges. Finally, Moghaddamfar et al. [16] found the number of spanning trees of the power graph associated with specific finite groups. They determined, up to isomorphism, the structure of a finite group G whose power graph has exactly n spanning trees, for $n < 5^3$. The author of the mentioned paper presented also a new characterization of the alternating group A_5 by tree-number of its power graph.

Suppose that G is a finite group and $x \in G$. If G is a finite group then it easy to prove that the power graph $\mathcal{P}(G)$ is a connected graph of diameter at most 2. The **degree** of x in $\mathcal{P}(G)$ can be calculated by $deg(x) = |\{g \in G \mid \langle x\rangle \leq \langle g\rangle \ or \ \langle g\rangle \leq \langle x\rangle\}|$. Suppose Γ is a graph with $V(\Gamma) = \{1, 2, \ldots, p\}$ and $\mathcal{F} = \{\Gamma_1, \ldots, \Gamma_p\}$ is a family of graphs such that $n_j = |V(\Gamma_j)|$, $1 \leq j \leq p$. Define $\Lambda = \Gamma[\Gamma_1, \ldots, \Gamma_p]$ to be a graph with vertex set $V(\Lambda) = \bigcup_{j=1}^{p} V(\Gamma_j)$ and edge set

$$E(\Lambda) = \big(\bigcup_{j=1}^{p} E(\Gamma_j)\big) \cup \big(\bigcup_{ij\in E(\Gamma)} \{uv; u \in V(\Gamma_i), v \in V(\Gamma_j)\}\big).$$

The graph Λ is called the **Γ-join of** $\mathcal{F}$ [3]. The set of all positive divisors of an integer n is denoted by $D(n)$. Our other notations are standard and can be taken from [14, 18].

2. Main Results

The aim of this section is to prove $\mathcal{P}(\mathbb{Z}_n) = K_{\phi(n)+1} + \Delta_n[K_{\phi(d_1)}, K_{\phi(d_2)}, \ldots, K_{\phi(d_p)}]$, where Δ_n is a graph with vertex and edge sets $V(\Delta_n) = \{d_i \mid 1, n \neq d_i|n, 1 \leq i \leq p\}$ and $E(\Delta_n) = \{d_id_j \mid d_i|d_j, 1 \leq i < j \leq p\}$, respectively. As a consequence it is proved that $Aut(\mathcal{P}(\mathbb{Z}_n)) \cong S_{\phi(n)+1} \times \prod_{1,n\neq d|n} S_{\phi(d)}$.

This proves a recent conjecture by Doostabadi et al. [5]. We start by computing the order and size of $M = \Delta_n[K_{\phi(d_1)}, K_{\phi(d_2)}, \ldots, K_{\phi(d_p)}]$.

Lemma 2.1. *$|V(M)| = n - \phi(n) - 1$ and $|E(M)| = \frac{1}{2}\sum_{1,n\neq d|n}(2d - \phi(d) - 3)\phi(d)$.*

Proof. Since $\sum_{d|n}\phi(d) = n$, $|V(M)| = \sum_{1,n\neq d|n}\phi(d) = n - \phi(n) - 1$. On the other hand,

$$\begin{aligned}\sum_{\substack{(d\neq d'),d|d'\\ 1,n\neq d,d'|n}}\phi(d)\phi(d') &= \sum_{1,n\neq d|n}\sum_{\substack{d'|d\\ d'\neq 1,d}}\phi(d)\phi(d')\\ &= \sum_{1,n\neq d|n}\phi(d)\sum_{1,d\neq d'|d}\phi(d')\\ &= \sum_{1,n\neq d|n}(d-\phi(d)-1)\phi(d).\end{aligned}$$

Therefore,

$$\begin{aligned}2|E(M)| &= \sum_{1,n\neq d|n}\phi(d)(\phi(d)-1) + 2\sum_{dd'\in E(\Delta_n)}\phi(d)\phi(d')\\ &= \sum_{1,n\neq d|n}\phi(d)^2 - \sum_{1,n\neq d|n}\phi(d) + 2\sum_{1,n\neq d|n}(d-\phi(d)-1)\phi(d)\\ &= 2\sum_{1,n\neq d|n}d\phi(d) - 3\sum_{1,n\neq d|n}\phi(d) - \sum_{1,n\neq d|n}\phi(d)^2\\ &= \sum_{1,n\neq d|n}(2d-\phi(d)-3)\phi(d),\end{aligned}$$

proving the result. □

Theorem 2.2. *$\mathcal{P}(\mathbb{Z}_n) = K_{\phi(n)+1} + \Delta_n[K_{\phi(d_1)}, K_{\phi(d_2)}, \ldots, K_{\phi(d_p)}]$.*

Proof. We first assume that the power graph $\mathcal{P}(\mathbb{Z}_n)$ has a subgraph isomorphic to

$$K_{\phi(n)+1} + \Delta_n[K_{\phi(d_1)}, K_{\phi(d_2)}, \ldots, K_{\phi(d_p)}].$$

Suppose $\{d_1, \ldots, d_p\} = D(n) - \{1, n\}$ and x, y are vertices in $\mathcal{P}(\mathbb{Z}_n)$. Clearly, if x and y are adjacent then $o(x)|o(y)$ or $o(y)|o(x)$. Thus, $\mathcal{P}(\mathbb{Z}_n)$ has complete subgroups of orders $\phi(d_i)$, $1 \leq i \leq p$. If for some i and j, $d_i|d_j$, then since $\mathbb{Z}_n$ is a cyclic group, all vertices of degree d_i and d_j are adjacent. On the other hand, all generators of $\mathbb{Z}_n$ together with identity element constitute a complete subgraph of order $\phi(n)+1$ that its vertices are adjacent to all other vertices of $\mathcal{P}(\mathbb{Z}_n)$. This proves that the power graph $\mathcal{P}(\mathbb{Z}_n)$ has a subgraph H isomorphic to $K_{\phi(n)+1} + \Delta_n[K_{\phi(d_1)}, K_{\phi(d_2)}, \ldots, K_{\phi(d_p)}]$.

To complete the proof, we calculate the number of vertices and edges of H. By Lemma 2.1, $|V(H)| = \phi(n)+1+|V(M)| = \phi(n)+1+n-\phi(n)-1 = n$ and $|E(H)| = \frac{\phi(n)(\phi(n)+1)}{2} + (\phi(n)+1)(n-$

$\phi(n)-1)+|E(M)|$. Therefore,

$$\begin{aligned} 2|E(H)| &= (2n-\phi(n)-1)\phi(n)+2(n-\phi(n)-1)+\sum_{1,n\neq d|n}(2d-\phi(d)-3)\phi(d)\\ &= (2n-\phi(n)-1)\phi(n)+2\sum_{1,n\neq d|n}\phi(d)+\sum_{1,n\neq d|n}(2d-\phi(d)-3)\phi(d)\\ &= (2n-\phi(n)-1)\phi(n)+\sum_{1,n\neq d|n}(2d-\phi(d)-1)\phi(d)\\ &= \sum_{d|n}(2d-\phi(d)-1)\phi(d). \end{aligned}$$

By [4, Corollary 4.3], $2|E(\mathcal{P}(\mathbb{Z}_n))|=\sum_{d|n}(2d-\phi(d)-1)\phi(d)$, which shows that $H=\mathcal{P}(\mathbb{Z}_n)$. Hence, $\mathcal{P}(\mathbb{Z}_n)=K_{\phi(n)+1}+\Delta_n[K_{\phi(d_1)},K_{\phi(d_2)},\ldots,K_{\phi(d_p)}]$ which completes the proof. □

By [4, Theorem 2.12], it is clear that the mentioned conjecture made by Doostabadi et al. [5] is incorrect, when n is prime power. In the next theorem this conjecture is proved for positive integer n such that n cannot be written as a prime power.

Theorem 2.3. *If n is not a prime power then $Aut(\mathcal{P}(\mathbb{Z}_n))\cong S_{\phi(n)+1}\times\prod_{1,n\neq d|n}S_{\phi(d)}$.*

Proof. It is well-known that for each graph Γ, $Aut(\Gamma)\cong Aut(\bar{\Gamma})$. Applying Theorem 2.2, we have:

$$\begin{aligned} Aut(\mathcal{P}(\mathbb{Z}_n)) &= Aut(\overline{\mathcal{P}(\mathbb{Z}_n)})\\ &= Aut(\overline{K_{\phi(n)+1}+M})\\ &= Aut(\overline{K_{\phi(n)+1}}\cup\overline{M})\\ &= Aut(\overline{K_{\phi(n)+1}})\times Aut(\overline{M}). \end{aligned}$$

Since $Aut(\overline{K_{\phi(n)+1}})=Aut(K_{\phi(n)+1})\cong S_{\phi(n)+1}$, $Aut(\mathcal{P}(\mathbb{Z}_n))\cong S_{\phi(n)+1}\times Aut(\overline{M})\cong S_{\phi(n)+1}\times Aut(M)$. So, it is enough to prove that $Aut(M)\cong\prod_{1,n\neq d|n}S_{\phi(d)}$. To do this, we first calculate the degree of each vertex $x\in K_{\phi(d)}$ in M. By the definition of M, we have:

$$\begin{aligned} deg_M(x) &= \phi(o(x))-1+\sum_{\substack{d|o(x)\\ 1,n,o(x)\neq d|n}}\phi(d)+\sum_{\substack{o(x)|d\\ 1,n,o(x)\neq d|n}}\phi(d)\\ &= \phi(o(x))-1+o(x)-\phi(o(x))-1+\sum_{\substack{o(x)|d\\ 1,n,o(x)\neq d|n}}\phi(d)\\ &= o(x)-2+\sum_{\substack{o(x)|d|n\\ 1,n,o(x)\neq d}}\phi(d). \end{aligned}$$

Define $H_d=K_{\phi(d)}$. We prove that for each automorphism $\alpha\in Aut(M)$, $\alpha(H_{d_i})=H_{d_i}$. To do this, we prove that there is no automorphism $\beta\in Aut(M)$ such that $\beta(H_{d_i})=H_{d_j}, i\neq j$. Choose $x\in H_{d_i}$ and $y\in H_{d_j}$, $i\neq j$. If $deg_M(x)\neq deg_M(y)$ then it is obvious that there is no automorphism β such that $\beta(x) = y$. If $deg_M(x) = deg_M(y)$ then

$$o(x) - 2 + \sum_{o(x)|d|n} \phi(d) = o(y) - 2 + \sum_{o(y)|d|n} \phi(d). \tag{2.1}$$

Without loss of generality, we can assume that $o(x) < o(y)$. So,

$$\sum_{o(x)|d|n} \phi(d) > \sum_{o(y)|d|n} \phi(d). \tag{2.2}$$

We consider two separate cases as follows:

Case 1: $o(x)|o(y)$. In this case, if $o(y) \mid d$ then $o(x) \mid d$ and so each summand in the right hand side of (2) is a summand of the left hand side of this equation. Thus the inequality (2.2) implies that there is a positive integer d such that $d \mid n$, $o(x) \mid d$ and $o(y) \nmid d$. Consider the complete subgraphs $A = K_{\phi(d)}$, $B = K_{\phi(o(x))}$ and $C = K_{\phi(o(y))}$. Then each vertex of A is adjacent to each vertex of B, but there is no edge connecting a vertex of A and a vertex of C. Hence there is no automorphism that sends x to y.

Case 2: $o(x) \nmid o(y)$. Suppose there is no d such that $d|n$ and $o(x)|d$. Then $\sum_{o(x)|d|n} \phi(d) = 0$. Apply inequality $o(x) < o(y)$ and Eq. (2.1) to deduce that $deg_M(x) \neq deg_M(y)$, which is impossible. Put $d = ko(x) \mid n$, where $k \neq 0, 1, t\dfrac{o(y)}{gcd(o(x), o(y))}$, for $t \geq 1$. If $o(y)|d$ then there exists k' such that $ko(x) = k'o(y)$, that means that $k\dfrac{o(x)}{gcd(o(x), o(y))} = k'\dfrac{o(y)}{gcd(o(x), o(y))}$. But $\dfrac{o(x)}{gcd(o(x), o(y))}$ and $\dfrac{o(y)}{gcd(o(x), o(y))}$ are coprime, which implies that $\dfrac{o(x)}{gcd(o(x), o(y))} \mid k'$. Hence,

$$k = \Big(\frac{k' gcd(o(x), o(y))}{o(x)}\Big)\Big(\frac{o(y)}{gcd(o(x), o(y))}\Big) = t\frac{o(y)}{gcd(o(x), o(y))},$$

a contradiction. Again, we consider the complete subgraphs $A = K_{\phi(d)}$, $B = K_{\phi(o(x))}$ and $C = K_{\phi(o(y))}$. Then each vertex of A is adjacent to each vertex of B, but there is no edge connecting a vertex of A and a vertex of C. Hence there is no automorphism that sends x to y. This completes our argument. □

Corollary 2.4. *The automorphism group of the power graph D_{2n} can be computed as follows:*

$$Aut(\mathcal{P}(D_{2n})) \cong \begin{cases} S_{n-1} \times S_n & n \text{ is a prime power} \\ \\ S_n \times \prod_{d|n} S_{\phi(d)} & \text{otherwise} \end{cases}$$

Proof. By [14, Proposition 7], $\mathcal{P}(D_{2n})$ is a union of $\mathcal{P}(\mathbb{Z}_n)$ and n copies of K_2 that share the identity element of D_{2n}. If n is prime power then $\mathcal{P}(D_{2n}) \cong K_{n-1} + K_1 + \overline{K_n}$ and so $Aut(\mathcal{P}(D_{2n})) \cong S_{n-1} \times S_n$. Otherwise, $\mathcal{P}(D_{2n}) \cong \mathcal{P}({\mathbb{Z}_n}^*) + K_1 + \overline{K_n}$ and we have $Aut(\mathcal{P}(D_{2n})) \cong S_n \times \prod_{d|n} S_{\phi(d)}$, proving the result. □

To describe our result, we compute the automorphism groups of $\mathcal{P}(G)$, for some special group G.

Example 2.5. In this example the automorphism groups of $\mathcal{P}(\mathbb{Z}_{pq})$, $\mathcal{P}(\mathbb{Z}_{pqr})$ and $\mathcal{P}(\mathbb{Z}_{p^2q^2})$ are calculated. We first assume that $n = pq$, $x \in H_p$ and $y \in H_q$. Then $deg_M(x) = p - 2$ and $deg_M(y) = q - 2$ and so $deg_M(x) \neq deg_M(y)$. Applying Theorem 2.3, we have $Aut(\mathcal{P}(\mathbb{Z}_{pq})) \cong$

$S_{\phi(pq)+1} \times S_{p-1} \times S_{q-1}$. Next, we suppose that $n = pqr$. Then $V(\Delta_n) = \{p, q, r, pq, pr, qr\}$ and $E(\Delta_n) = \{(p, pq), (p, pr), (q, pq), (q, qr), (r, pr), (r, qr)\}$. By Theorem 2.2,

$$\mathcal{P}(\mathbb{Z}_{pqr}) = K_{\phi(pqr)+1} + \Delta_n[H_p, H_q, H_r, H_{pq}, H_{pr}, H_{qr}].$$

Choose $x \in H_p$, $y \in H_q$, $z \in H_r$, $u \in H_{pq}$, $v \in H_{pr}$ and $w \in H_{qr}$. Then,

$$\begin{aligned}
deg_M(x) &= p - 2 + pq - p - q + 1 + pr - p - r + 1 = pq + pr - p - q - r,\\
deg_M(y) &= q - 2 + pq - p - q + 1 + qr - q - r + 1 = pq + qr - p - q - r,\\
deg_M(z) &= r - 2 + pr - p - r + 1 + qr - q - r + 1 = pr + qr - p - q - r,\\
deg_M(u) &= pq - 2, deg_M(v) = pr - 2, deg_M(w) = qr - 2.
\end{aligned}$$

Therefore by Theorem 2.3,

$$Aut(\mathcal{P}(\mathbb{Z}_{pqr})) \cong S_{\phi(pqr)+1} \times S_{p-1} \times S_{q-1} \times S_{r-1} \times S_{\phi(pq)} \times S_{\phi(pr)} \times S_{\phi(qr)}.$$

Finally, we consider the case that $n = p^2q^2$. Then,

$$\begin{aligned}
V(\Delta_n) &= \{p, p^2, q, q^2, pq, pq^2, p^2q\}\\
E(\Delta_n) &= \{(p, p^2), (p, pq), (p, pq^2), (p, p^2q), (q, q^2), (q, pq),\\
&\quad (q, pq^2), (q, p^2q), (p^2, p^2q), (q^2, pq^2), (pq, pq^2), (pq, p^2q)\}.
\end{aligned}$$

Again by Theorem 2.2,

$$\mathcal{P}(\mathbb{Z}_{p^2q^2}) = K_{\phi(p^2q^2)+1} + \Delta_n[H_p, H_{p^2}, H_q, H_{q^2}, H_{pq}, H_{pq^2}, H_{p^2q}].$$

Choose $x \in H_p$, $y \in H_{p^2}$, $z \in H_q$, $u \in H_{q^2}$, $v \in H_{pq}$, $w \in H_{pq^2}$ and $r \in H_{p^2q}$. Then,

$$\begin{aligned}
deg_M(x) &= pq^2 + p^2q - pq - q^2 - 1,\\
deg_M(y) &= p^2q - pq + p - 2,\\
deg_M(z) &= pq^2 + p^2q - p^2 - pq - 1,\\
deg_M(u) &= pq^2 - pq + q - 2,\\
deg_M(v) &= pq^2 + p^2q - pq - p^2 - q^2 + p + q - 2,\\
deg_M(w) &= pq^2 - 2, deg_M(r) = p^2q - 2.
\end{aligned}$$

Since degrees are different, by Theorem 2.3,

$$Aut(\mathcal{P}(\mathbb{Z}_{p^2q^2})) \cong S_{\phi(p^2q^2)+1} \times S_{p-1} \times S_{\phi(p^2)} \times S_{q-1} \times S_{\phi(q^2)} \times S_{\phi(pq)} \times S_{\phi(pq^2)} \times S_{\phi(p^2q)}.$$

3. Concluding Remarks

The semidihedral group SD_{8n} and dicyclic group T_{4n} can be presented as follows:

$$\begin{aligned}
SD_{8n} &= \ < a, b | a^{4n} = b^2 = 1, bab = a^{2n-1} >,\\
T_{4n} &= \ < a, b | a^{2n} = 1, a^n = b^2, b^{-1}ab = a^{-1} > .
\end{aligned}$$

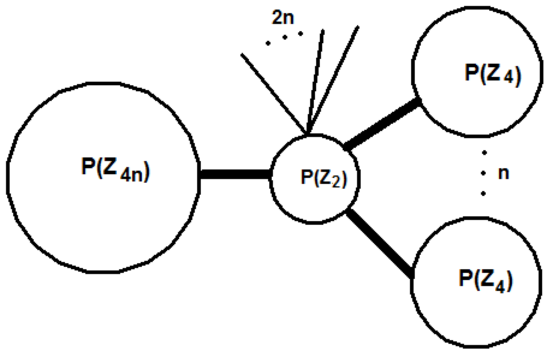

Figure 1. The Power Graph of SD_{8n}.

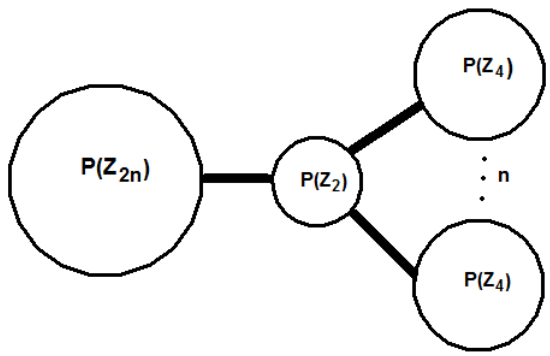

Figure 2. The Power Graph of T_{4n}.

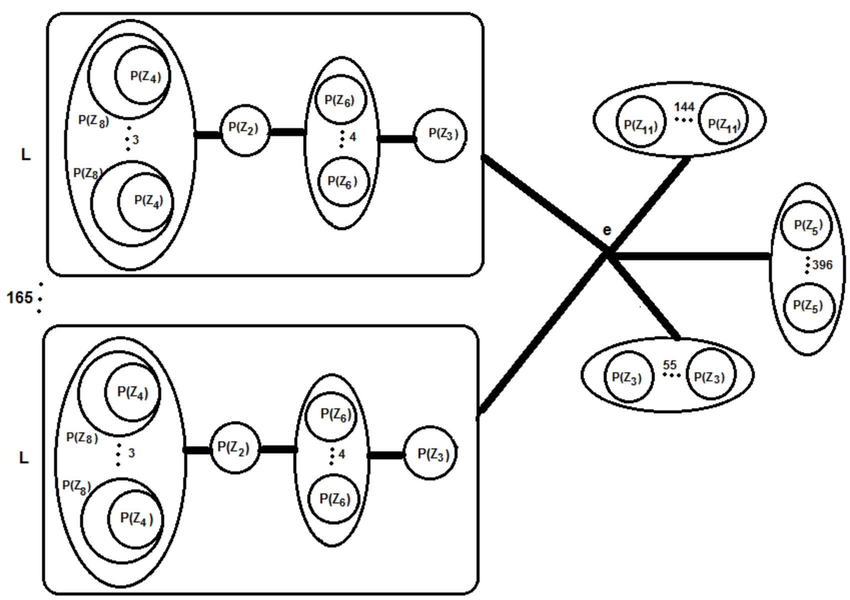

Figure 3. The Power Graph of M_{11}.

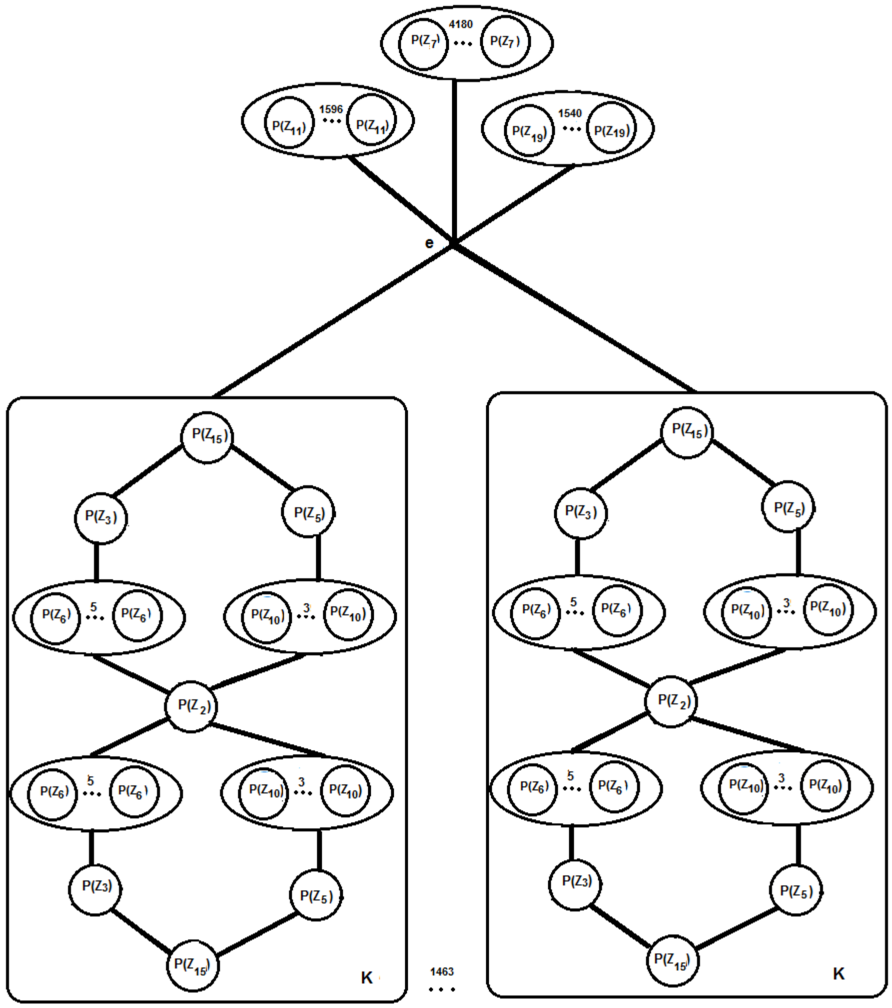

FIGURE 4. The Power Graph of J_1.

In this section, we apply similar methods as Theorem 2.2 to obtain the graph structure of $\mathcal{P}(G)$, where G is isomorphic to SD_{8n}, T_{4n}, the Mathieu group M_{11} or the Janko group J_1. The automorphism groups of these graphs can be computed in a similar way as those of the cyclic groups.

The power graph $\mathcal{P}(SD_{8n})$ is a union of $\mathcal{P}(\mathbb{Z}_{4n})$, n copies of $\mathcal{P}(\mathbb{Z}_4)$ that share an edge and $2n$ copies of $\mathcal{P}(\mathbb{Z}_2)$, all of them are connected to each other in the identity element of SD_{8n}, as shown in Figure 1.

The power graph $\mathcal{P}(T_{4n})$ can be constructed in a similar way as a union of $\mathcal{P}(\mathbb{Z}_{2n})$ and n copies of $\mathcal{P}(\mathbb{Z}_4)$ that share an edge, all connected to each other in the identity element of T_{4n}, as shown in Figure 2.

In what follows, we explain the power graph of the sporadic groups M_{11}, Figure 3, and J_1, Figure 4. The power graph $\mathcal{P}(M_{11})$ has exactly 7920 vertices. It can be constructed from 165 copies of a graph L, 55 copies of $\mathcal{P}(\mathbb{Z}_3)$, 396 copies of $\mathcal{P}(\mathbb{Z}_5)$ and 144 copies of $\mathcal{P}(\mathbb{Z}_{11})$, all connected to each other in the identity element of M_{11}.

The Janko group J_1 has exactly 175560 elements and its power graph is a union of 1463 copies of a graph K, 1540 copies of $\mathcal{P}(\mathbb{Z}_{19})$, 1596 copies of $\mathcal{P}(\mathbb{Z}_{11})$ and 4180 copies of $\mathcal{P}(\mathbb{Z}_7)$, all connected to each other in the identity element of J_1.

We end this paper with the following open question:

Question 3.1. *What is the automorphism group of $\mathcal{P}(G)$, where G is a sporadic group?*

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Zeinab Mehranian

Department of Mathematics, University of Qom, Qom, I. R. Iran

Ahmad Gholami

Department of Mathematics, University of Qom, Qom, I. R. Iran

Ali Reza Ashrafi

Department of Pure Mathematics, Faculty of Mathematical Sciences, University of Kashan, P. O. Box 87317–51116, Kashan, I. R. Iran

Email: `ashrafi@kashanu.ac.ir`