www.theoryofgroups.ir

International Journal of Group Theory
ISSN (print): 2251-7650, ISSN (on-line): 2251-7669
Vol. 2 No. 3 (2013), pp. 19-38.

www.ui.ac.ir

THE FISCHER-CLIFFORD MATRICES OF THE INERTIA GROUP $2^7{:}O_6^-(2)$ OF A MAXIMAL SUBGROUP $2^7{:}Sp_6(2)$ IN $Sp_8(2)$

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Communicated by Jamshid Moori

Abstract. The subgroups of symplectic groups which fix a non-zero vector of the underlying symplectic space are called *affine subgroups.* The split extension group $A(4) \cong 2^7{:}Sp_6(2)$ is the affine subgroup of the symplectic group $Sp_8(2)$ of index 255. In this paper, we use the technique of the Fischer-Clifford matrices to construct the character table of the inertia group $2^7{:}O_6^-(2)$ of $A(4)$ of index 28.

1. Introduction

Let $\overline{G} = N{:}G$ be a split extension of N by G. Then for $\theta \in Irr(N)$, define

$$\overline{H} = \{x \in \overline{G} | \theta^x = \theta\} = I_{\overline{G}}(\theta)$$
$$H = \{g \in G | \theta^g = \theta\} = I_G(\theta).$$

Since $I_{\overline{G}}(\theta)$ is the stabilizer of θ in the action of $\overline{G}$ on $Irr(N)$, we have that $I_{\overline{G}}(\theta)$ is a subgroup of $\overline{G}$ and N is normal in $I_{\overline{G}}(\theta)$. Also $[\overline{G}, I_{\overline{G}}(\theta)]$ is the size of the orbit containing θ. Then it can be shown that $\overline{H} = N{:}H$, where $\overline{H}$ is the inertia group of θ in $\overline{G}$. The inertia factor $\overline{H}/N \cong H$ can be regarded as the inertia group of θ in the factor group $\overline{G}/N \cong G$.

In this paper we are concerned with the inertia group $2^7{:}O_6^-(2)$ in $A(4)$, the affine subgroup of $Sp_8(2)$. $A(4)$ is the maximal subgroup of $Sp_8(2)$ fixing the non-zero vector e_1 in $V_8(2)$, where $V_8(2)$ is the

MSC(2010): Primary: 20C15; Secondary: 20C40.

Keywords: Fischer-Clifford matrices, inertia group, split extension, character table, coset analysis.

Received: 14 November 2012, Accepted: 30 November 2012.

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vector space of dimension 8 over $GF(2)$. We obtain from Theorem 6.2.6 and Lemma 6.2.5 (see [1]) that

$$A(4) = [Sp_8(2)]_{e_1} = P(4){:}H = 2^7{:}Sp_6(2).$$

Ali [1] constructed both of 2^7 and $Sp_6(2)$ in terms of 8×8 matrices inside $Sp_8(2)$ and then act $Sp_6(2)$ on 2^7 by conjugation to represent $Sp_6(2)$ in terms of 7×7 matrices over $GF(2)$. The group $2^7{:}Sp_6(2)$ acts on $Irr(2^7)$ to produce four inertia groups $\bar{H}_i = 2^7{:}H_i$ of indices 1, 36, 28 and 63 in $2^7{:}Sp_6(2)$, respectively and where $i \in \{1, 2, 3, 4\}$. The groups are $2^7{:}Sp_6(2)$, $2^7{:}S_8$, $2^7{:}O_6^-(2)$ and $2^7{:}(2^5{:}S_6)$, where the inertia factor groups H_i, that is, S_8, $O_6^-(2)$ and $2^5{:}S_6$ are maximal subgroups of $Sp_6(2)$. Note that $O_6^-(2) \cong U_4(2){:}2$. The reader is referred to Ali [1] regarding the above mentioned inertia groups which are all split extensions and sit maximally in $2^7{:}Sp_6(2)$.

Ali [1] has already calculated the character table of $2^7{:}Sp_6(2)$ by the method of Fischer-Clifford matrices. In this paper, the conjugacy classes and the Fischer-Clifford matrices of $2^7{:}O_6^-(2)$ will be computed. We shall use the technique of the Fischer-Clifford matrices to construct the character table of $2^7{:}O_6^-(2)$. We shall use the properties of the Fischer-Clifford matrices and additional information, which are discussed in Chapter 5 (see [13]), to construct their entries. Most of the Fischer-Clifford matrices have several candidates satisfying the properties discussed in Chapter 5 [13] and therefore we used the additional information and methods in the elimination process. The complete fusion of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$ will be determined. Motivation for the problem came from the Ph.D thesis of Ali [1].

2. **Theory of Fischer-Clifford Matrices**

Since the character table of $2^7{:}O_6^-(2)$ will be constructed by means of its Fischer-Clifford matrices we will give a brief theoretical background of this technique. Here we will follow the work of Whitley [16] and Mpono [13].

Let $\overline{G} = N.G$ be an extension of N by G and $\theta \in Irr(N)$. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$ for $g \in \overline{G}$ and $n \in N$ and $\theta^g \in Irr(N)$. Let $\overline{H}$ be the inertia group of θ in $\overline{G}$ then N is normal in $\overline{H}$. We say that θ is extendible to $\overline{H}$ if there exists $\phi \in Irr(\overline{H})$ such that $\phi \downarrow_N = \theta$. If θ is extendible to $\overline{H}$, then by Gallagher [9], we have

$$\left\{\psi | \psi \in Irr(\overline{H}), < \psi \downarrow_N, \theta > \neq 0\right\} = \left\{\beta\phi | \beta \in Irr(\overline{H}/N)\right\}.$$

Let $\overline{G}$ has the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$, $\overline{H}_i = I_{\overline{G}}(\phi_i)$, $1 \leq i \leq t$, $\phi_i \in Irr(\overline{H}_i)$ be an extension of θ_i to $\overline{H}_i$ and $\beta \in Irr(\overline{H}_i)$ such that $N \subseteq ker(\beta)$. Then it can be shown that

$$Irr(\overline{G}) = \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \mid \beta \in Irr(\overline{H}_i), N \subseteq ker(\beta)\} = \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \mid \beta \in Irr(\overline{H}_i/N)\}$$

Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H}_i$.

Let H_i be the inertia factor group and ϕ_i be an extension of θ_i to $\overline{H}_i$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H}_1 = \overline{G}$ and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. We define

$$R(g) = \{(i, y_k) \mid 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\}$$

and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Then we define the Fischer-Clifford matrix $M(g)$ by $M(g) = (a^j_{(i,y_k)})$, where

$$a^j_{(i,y_k)} = \sum_{l}{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\phi_i(y_{l_k})\,,$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by $\begin{bmatrix} C_1(g)\,M_1(g) \\ C_2(g)\,M_2(g) \\ \vdots \\ C_t(g)\,M_t(g) \end{bmatrix}$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$ in G. We can also observe that the number of irreducible characters of $\overline{G}$ is the sum of the number of irreducible characters of the inertia factors H_i's. The group $\overline{G} = 2^7{:}O_6^-(2)$ is a split extension with 2^7 abelian and therefore by Mackey's theorem [9] each irreducible character of 2^7 can be extended to its inertia group in $\overline{G}$. Hence by the above theoretical outline we can fully determine the character table of $2^7{:}O_6^-(2)$.

3. The Conjugacy Classes of $2^7{:}O_6^-(2)$

In this section we use the method of coset analysis to determine the conjugacy classes of elements of $2^7{:}O_6^-(2)$. We refer readers to [1],[10],[13] and [16] for full details and background material regarding the method of coset analysis. Most of the information, which involved the conjugacy classes and permutation characters, were obtained by using direct computations in MAGMA [3] and therefore the classes and permutation characters may be represented differently in ATLAS [5]. We generated $O_6^-(2)$ by two elements $g_1 \in 4A$ and $g_2 \in 6A$ of $Sp_6(2)$. Ali [1] constructed $Sp_6(2)$ in terms of 7×7 matrices over $GF(2)$. Here $4A$ and $6A$ are conjugacy classes of elements of $Sp_6(2)$ and g_1 and g_2 are given by :

$$g_1 = \begin{pmatrix} 1&1&1&1&1&1&1\\ 0&0&1&1&1&1&1\\ 0&1&1&1&0&0&1\\ 0&1&1&0&1&1&0\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 0&1&0&1&0&0&1\\ 1&1&1&1&1&0&1\\ 1&0&0&0&0&1&1\\ 0&0&0&0&1&1&0\\ 0&1&1&0&0&0&0\\ 1&1&0&1&0&1&1\\ 1&1&0&1&0&0&0 \end{pmatrix}$$

The elements g_1 and g_2 are of order 4 and 6, respectively. We obtained the conjugacy classes of $O_6^-(2)$ using MAGMA [3] and are represented in the Table 1. The class representatives of each class $[g]_{O_6^-(2)}$ of $O_6^-(2)$ are given in terms of 7×7 matrices over $GF(2)$.

Table 1. The conjugacy classes of $O_6^-(2)$

$[g]_{O_6^-(2)}$	Class representative	$\lvert[g]_{O_6^-(2)}\rvert$	$[g]_{O_6^-(2)}$	Class representative	$\lvert[g]_{O_6^-(2)}\rvert$
$1A$	$\begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	1	$2A$	$\begin{pmatrix} 1&1&1&1&0&1&1\\ 0&1&0&0&0&0&0\\ 0&1&0&1&0&1&1\\ 0&1&1&0&0&1&1\\ 0&1&1&1&1&1&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	36
$2B$	$\begin{pmatrix} 1&1&0&1&1&0&0\\ 0&0&0&1&1&0&1\\ 0&0&1&0&0&0&0\\ 0&1&0&0&1&0&1\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	45	$2C$	$\begin{pmatrix} 1&1&1&1&0&1&1\\ 0&0&1&1&0&1&0\\ 0&0&1&0&0&0&1\\ 0&1&1&0&0&1&1\\ 0&0&0&0&1&0&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	270
$2D$	$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&0&0&1&1&0&1\\ 0&1&0&1&0&1&1\\ 0&0&1&1&1&1&0\\ 0&1&1&1&1&1&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	540	$3A$	$\begin{pmatrix} 1&1&0&0&1&1&0\\ 1&0&1&0&1&0&1\\ 0&0&0&0&1&1&0\\ 1&1&0&1&1&1&1\\ 0&1&1&1&0&1&1\\ 0&1&0&1&1&1&0\\ 0&1&0&1&1&0&1 \end{pmatrix}$	80
$3B$	$\begin{pmatrix} 1&1&1&1&1&1&1\\ 0&1&0&0&1&0&0\\ 0&1&0&1&0&1&1\\ 0&1&1&0&0&1&1\\ 0&1&1&1&0&1&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	240	$3C$	$\begin{pmatrix} 0&1&1&1&0&1&0\\ 0&1&0&0&1&0&0\\ 1&0&1&1&0&1&1\\ 1&0&0&0&0&1&1\\ 0&1&1&1&0&1&1\\ 0&1&1&0&1&1&1\\ 0&1&1&0&1&0&0 \end{pmatrix}$	480
$4A$	$\begin{pmatrix} 0&0&0&1&1&1&1\\ 1&0&1&0&1&0&1\\ 0&1&0&1&0&1&0\\ 1&0&0&0&1&1&0\\ 0&1&1&1&1&1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	540	$4B$	$\begin{pmatrix} 1&1&1&1&1&1&1\\ 0&1&0&0&1&0&0\\ 0&1&1&1&1&0&0\\ 0&1&0&0&0&0&1\\ 0&1&0&1&0&0&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	540

Table 1 (continued)

$[g]_{O_6^-(2)}$	Class representative	$\lvert[g]_{O_6^-(2)}\rvert$	$[g]_{O_6^-(2)}$	Class representative	$\lvert[g]_{O_6^-(2)}\rvert$
$4C$	$\begin{pmatrix} 0&1&0&0&0&1&0\\ 1&1&1&1&0&0&1\\ 0&1&0&1&0&1&0\\ 1&1&0&1&0&1&0\\ 0&1&1&1&1&1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	1620	$4D$	$\begin{pmatrix} 0&1&1&0&1&0&1\\ 1&1&0&1&1&1&0\\ 0&1&1&1&1&0&1\\ 1&1&0&1&0&1&0\\ 0&1&0&1&0&0&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	3240
$5A$	$\begin{pmatrix} 0&1&1&0&1&0&1\\ 1&0&0&1&0&0&1\\ 0&0&0&1&1&0&0\\ 0&1&0&1&0&1&0\\ 1&1&0&1&0&0&1\\ 1&1&1&0&1&1&1\\ 1&1&1&0&1&0&0 \end{pmatrix}$	5184	$6A$	$\begin{pmatrix} 1&1&0&1&1&0&0\\ 1&0&1&0&0&0&1\\ 1&1&1&1&0&0&0\\ 1&0&0&0&0&1&1\\ 0&0&1&1&0&0&0\\ 1&0&0&0&0&1&0\\ 1&0&0&0&0&0&1 \end{pmatrix}$	720
$6B$	$\begin{pmatrix} 1&0&1&0&1&1&1\\ 0&1&1&0&0&1&0\\ 0&0&1&0&1&0&0\\ 0&1&0&0&0&0&1\\ 0&1&1&1&1&1&1\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	1440	$6C$	$\begin{pmatrix} 0&0&0&1&0&1&1\\ 1&1&0&1&1&1&0\\ 1&0&1&0&0&0&0\\ 0&1&1&1&0&0&1\\ 0&1&1&1&1&1&1\\ 1&0&0&0&1&1&1\\ 1&0&0&0&1&0&0 \end{pmatrix}$	1440
$6D$	$\begin{pmatrix} 0&0&0&1&0&1&1\\ 0&1&0&1&1&1&0\\ 0&1&0&0&0&0&1\\ 0&1&1&1&0&0&1\\ 1&0&0&0&1&0&1\\ 1&1&1&0&1&1&0\\ 1&1&1&0&1&0&1 \end{pmatrix}$	1440	$6E$	$\begin{pmatrix} 0&1&1&1&1&1&0\\ 0&0&1&1&1&1&1\\ 0&0&1&0&1&0&0\\ 0&0&0&1&1&0&0\\ 0&1&1&1&1&1&1\\ 1&0&0&0&1&1&1\\ 1&0&0&0&1&0&0 \end{pmatrix}$	1440
$6F$	$\begin{pmatrix} 1&0&1&0&1&1&1\\ 1&1&1&0&1&1&0\\ 0&0&1&0&1&0&0\\ 1&1&0&0&1&0&1\\ 0&1&1&1&1&1&1\\ 0&1&0&1&1&1&0\\ 0&1&0&1&1&0&1 \end{pmatrix}$	2160	$6G$	$\begin{pmatrix} 0&0&1&0&0&1&1\\ 0&1&0&1&1&1&0\\ 0&1&1&1&0&0&1\\ 0&1&0&0&0&0&1\\ 1&0&0&0&1&0&1\\ 1&1&0&1&1&1&0\\ 1&1&0&1&1&0&1 \end{pmatrix}$	4320
$8A$	$\begin{pmatrix} 0&0&1&1&1&0&1\\ 1&1&1&1&0&0&1\\ 0&0&0&0&1&1&0\\ 1&1&0&1&0&1&0\\ 0&1&1&1&1&1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}$	6480	$9A$	$\begin{pmatrix} 0&0&0&0&1&0&1\\ 0&0&0&1&1&0&1\\ 1&1&0&0&0&0&1\\ 1&0&1&1&0&1&0\\ 1&1&1&0&0&0&0\\ 1&1&0&1&1&1&0\\ 1&1&0&1&1&0&1 \end{pmatrix}$	5760
$10A$	$\begin{pmatrix} 1&1&0&0&0&1&1\\ 1&0&1&0&1&0&1\\ 1&0&0&0&1&1&0\\ 0&0&1&0&1&0&1\\ 0&1&1&1&1&1&0\\ 1&0&1&0&0&0&1\\ 1&0&1&0&0&1&0 \end{pmatrix}$	5184	$12A$	$\begin{pmatrix} 1&1&1&1&0&1&1\\ 1&0&0&0&1&1&1\\ 1&0&0&0&0&1&1\\ 0&0&1&0&0&0&0\\ 0&1&0&1&0&0&1\\ 1&0&0&1&1&0&0\\ 1&0&0&1&1&1&1 \end{pmatrix}$	4320
$12B$	$\begin{pmatrix} 0&0&1&0&1&1&0\\ 1&0&0&1&1&0&0\\ 0&1&0&1&0&1&1\\ 1&0&1&1&0&1&1\\ 0&1&1&1&0&1&1\\ 1&0&0&0&1&1&1\\ 1&0&0&0&1&0&0 \end{pmatrix}$	4320			

$O_6^-(2)$ has 25 conjugacy classes and under the action of $O_6^-(2)$ on 2^7 we obtain six orbits of lengths 1, 1, 27, 27, 36 and 36. The point stabilizers for the orbits of lengths 1, 27 and 36 are $O_6^-(2)$, $2^4{:}S_5$ and $S_6 \times 2$ of indices 1, 27 and 36, respectively in $O_6^-(2)$. We used Programme A (Ali [1]) in MAGMA [3] to compute the orbit lengths and the corresponding stabilizers. The Programme A was originally developed in CAYLEY [4] by Mpono [13] and later the programme was adapted for MAGMA by Ali [1]. Let $\chi(O_6^-(2)|2^7)$ be the permutation character of $O_6^-(2)$ on 2^7. We also let $\chi(O_6^-(2)|2^4{:}S_5)$ and $\chi(O_6^-(2)|S_6 \times 2)$ be the permutation characters of $O_6^-(2)$ on $2^4{:}S_5$ and $S_6 \times 2$, respectively. The permutation characters $\chi(O_6^-(2)|2^4{:}S_5) = 1a + 6b + 20c$ and $\chi(O_6^-(2)|S_6 \times 2) = 1a + 15b + 20c$ are written in terms of the irreducible characters of $O_6^-(2)$ and are computed directly using the character tables of the point stabilizers together with the fusion maps of $2^4{:}S_5$ and $S_6 \times 2$ into $O_6^-(2)$.

We obtain that $\chi(O_6^-(2)|2^7) = I_{O_6^-(2)}^{O_6^-(2)} + I_{O_6^-(2)}^{O_6^-(2)} + I_{2^4:S_5}^{O_6^-(2)} + I_{2^4:S_5}^{O_6^-(2)} + I_{S_6\times 2}^{O_6^-(2)} + I_{S_6\times 2}^{O_6^-(2)}$, where $I_{2^4:S_5}^{O_6^-(2)}$ and $I_{S_6\times 2}^{O_6^-(2)}$ are the identity characters of $2^4{:}S_5$ and $S_6 \times 2$ respectively, induced to $O_6^-(2)$. We note that $I_{2^4:S_5}^{O_6^-(2)} = \chi(O_6^-(2)|2^4{:}S_5)$ and $I_{S_6\times 2}^{O_6^-(2)} = \chi(O_6^-(2)|S_6 \times 2)$.
Therefore

$$\begin{aligned}\chi(O_6^-(2)|2^7) &= I_{O_6^-(2)}^{O_6^-(2)} + I_{O_6^-(2)}^{O_6^-(2)} + I_{2^4:S_5}^{O_6^-(2)} + I_{2^4:S_5}^{O_6^-(2)} + I_{S_6\times 2}^{O_6^-(2)} + I_{S_6\times 2}^{O_6^-(2)} \\ &= 1a + 1a + 1a + 6b + 20c + 1a + 6b + 20c + 1a + 15b + 20c + 1a + 15b + 20c \\ &= 6 \times 1a + 2 \times 6b + 2 \times 15b + 4 \times 20c\end{aligned}$$

The values of $\chi(O_6^-(2)|2^7)$ on the different classes of $O_6^-(2)$ determine the number k of fixed points of each $g \in O_6^-(2)$ in 2^7. These values of the k's will enable us to calculate the conjugacy classes of $2^7{:}O_6^-(2)$ and are listed in Table 2.

TABLE 2. The values of $\chi(O_6^-(2)|2^7)$ on the different classes of $O_6^-(2)$

$[g]_{O_6^-(2)}$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$5A$
$\chi(O_6^-(2)\|O_6^-(2))$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(O_6^-(2)\|O_6^-(2))$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(O_6^-(2)\|2^4{:}S_5)$	27	15	3	7	3	0	9	0	1	3	5	1	2
$\chi(O_6^-(2)\|2^4{:}S_5)$	27	15	3	7	3	0	9	0	1	3	5	1	2
$\chi(O_6^-(2)\|S_6 \times 2)$	36	16	12	8	4	0	6	3	6	0	2	2	1
$\chi(O_6^-(2)\|S_6 \times 2)$	36	16	12	8	4	0	6	3	6	0	2	2	1
l	128	64	32	32	16	2	32	8	16	8	16	8	8

Table 2 (continued)

$[g]_{O_6^-(2)}$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$8A$	$9A$	$10A$	$12A$	$12B$
$\chi(O_6^-(2)\|O_6^-(2))$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(O_6^-(2)\|O_6^-(2))$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(O_6^-(2)\|2^4{:}S_5)$	0	3	0	0	3	1	0	1	0	0	0	1
$\chi(O_6^-(2)\|2^4{:}S_5)$	0	3	0	0	3	1	0	1	0	0	0	1
$\chi(O_6^-(2)\|S_6 \times 2)$	0	0	1	3	4	2	1	0	0	1	0	0
$\chi(O_6^-(2)\|S_6 \times 2)$	0	0	1	3	4	2	1	0	0	1	0	0
l	2	8	4	8	16	8	4	4	2	4	2	4

The values of k enabled us to determine the number f_j of orbits Q_i's, $1 \leq i \leq k$ which have fused together under the action of $C_{O_6^-(2)}(g)$, for each class representative $g \in O_6^-(2)$, to form one orbit $\triangle_f$. We used Programme A written in MAGMA [3] due to Ali [1] to calculate these f_j's. We obtained that $2^7{:}O_6^-(2)$ has exactly 130 conjugacy classes of elements. The values of the f_j's, the length of each class $[x]_{2^7:O_6^-(2)}$ and its corresponding centralizer $C_{2^7:O_6^-(2)}(x)$ are listed in Table 3. We also list the d_j's where d_jg is a representative of the $\triangle_f$. By Theorem 2.3.10 and Remark 2.3.11 (see [13]), we have

$$o(dg) = \begin{cases} m & \text{if} \quad w = 1_N \\ 2m & \text{otherwise} \end{cases},$$

for the class representative $dg \in \overline{G}$ where $d \in 2^7$, $g \in O_6^-(2)$ and $o(g) = m$. The above result together with Programme B (Ali [1]) developed in MAGMA [3] enabled us to compute the orders of the various class representatives of $2^7{:}O_6^-(2)$. Table 3 contains all the relevant information regarding the conjugacy classes of $2^7{:}O_6^-(2)$.

TABLE 3. The conjugacy classes of elements of $G = 2^7{:}O_6^-(2)$

$[g]_{O_6^-(2)}$	k	f_j	d_j	w	$[x]_{2^7:O_6^-(2)}$	$\|[x]_{2^7:O_6^-(2)}\|$	$\|C_{2^7:O_6^-(2)}(x)\|$
$1A$	128	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$1A$	1	6635520
		$f_2 = 1$	$(0,0,0,0,0,1,1)$	$(0,0,0,0,0,1,1)$	$2A$	1	6635520
		$f_3 = 27$	$(1,1,1,0,1,1,1)$	$(1,1,1,0,1,1,1)$	$2B$	27	245760
		$f_4 = 27$	$(1,1,1,1,1,1,0)$	$(1,1,1,1,1,1,0)$	$2C$	27	245760
		$f_5 = 36$	$(1,1,1,1,1,1,1)$	$(1,1,1,1,1,1,1)$	$2D$	36	184320
		$f_6 = 36$	$(1,1,0,1,1,1,1)$	$(1,1,0,1,1,1,1)$	$2E$	36	184320
$2A$	64	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2F$	72	92160
		$f_2 = 1$	$(0,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$2G$	72	92160
		$f_3 = 6$	$(1,1,1,1,0,1,0)$	$(0,1,1,1,1,1,0)$	$4A$	432	15360
		$f_4 = 6$	$(1,1,1,0,1,1,1)$	$(0,1,1,1,1,1,0)$	$4B$	432	15360
		$f_5 = 10$	$(1,0,1,1,0,1,1)$	$(0,1,1,1,1,1,0)$	$4C$	720	9216
		$f_6 = 10$	$(1,1,1,1,0,1,1)$	$(0,1,1,1,1,1,0)$	$4D$	720	9216
		$f_7 = 15$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2H$	1080	6144
		$f_8 = 15$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$2I$	1080	6144

Table 3(continued)

$[g]_{O_6^-(2)}$	k	f_j	d_j	w	$[x]_{2^7:O_6^-(2)}$	$\|[x]_{2^7:O_6^-(2)}\|$	$\|C_{2^7:O_6^-(2)}(x)\|$
$2B$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2J$	180	36864
		$f_2=1$	$(0,1,0,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$2K$	180	36864
		$f_3=3$	$(0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2L$	540	12288
		$f_4=3$	$(0,0,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$2M$	540	12288
		$f_5=12$	$(1,1,1,1,1,1,1)$	$(0,1,0,0,1,1,1)$	$4E$	2160	3072
		$f_6=12$	$(1,1,1,1,1,0,1)$	$(0,0,0,1,0,1,0)$	$4F$	2160	3072
$2C$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2N$	1080	6144
		$f_2=1$	$(0,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2O$	1080	6144
		$f_3=1$	$(1,0,1,1,1,0,1)$	$(0,0,1,1,1,0,0)$	$4G$	1080	6144
		$f_4=1$	$(1,0,1,1,1,1,0)$	$(0,0,1,1,1,0,0)$	$4H$	1080	6144
		$f_5=3$	$(1,1,0,1,1,1,1)$	$(0,0,1,1,1,0,0)$	$4I$	3240	2048
		$f_6=3$	$(1,1,1,1,1,1,1)$	$(0,0,1,1,1,0,0)$	$4J$	3240	2048
		$f_7=3$	$(0,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$2P$	3240	2048
		$f_8=3$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$2Q$	3240	2048
		$f_9=8$	$(1,1,1,1,1,1,0)$	$(0,0,1,0,0,1,1)$	$4K$	8640	768
		$f_{10}=8$	$(1,1,1,1,1,0,1)$	$(0,0,1,0,0,1,1)$	$4L$	8640	768
$2D$	16	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2R$	4320	1536
		$f_2=1$	$(1,0,1,1,0,1,0)$	$(0,0,0,0,0,0,0)$	$2S$	4320	1536
		$f_3=1$	$(1,1,1,1,0,1,1)$	$(0,0,1,0,0,1,0)$	$4M$	4320	1536
		$f_4=1$	$(1,1,1,0,0,1,1)$	$(0,0,1,0,0,1,0)$	$4N$	4320	1536
		$f_5=3$	$(1,1,1,1,1,1,1)$	$(0,0,1,1,0,0,1)$	$4O$	12960	512
		$f_6=3$	$(1,0,1,1,1,1,1)$	$(1,0,0,0,0,0,0)$	$4P$	12960	512
		$f_7=3$	$(1,1,1,1,0,1,0)$	$(1,0,1,1,0,0,1)$	$4Q$	12960	512
		$f_8=3$	$(1,1,1,1,1,0,1)$	$(1,0,1,0,0,1,0)$	$4R$	12960	512
$3A$	2	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3A$	5120	1296
		$f_2=1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,1,1)$	$6A$	5120	1296
$3B$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3B$	960	6912
		$f_2=1$	$(1,1,1,1,0,0,0)$	$(0,0,0,0,0,1,1)$	$6B$	960	6912
		$f_3=6$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,1,0,1)$	$6C$	5760	1152
		$f_4=6$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,1,1,0)$	$6D$	5760	1152
		$f_5=9$	$(1,1,0,1,1,1,1)$	$(1,0,0,1,1,0,1)$	$6E$	8640	768
		$f_6=9$	$(1,1,1,1,1,1,1)$	$(1,0,1,1,1,0,1)$	$6F$	8640	768
$3C$	8	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3C$	7680	864
		$f_2=1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,1,1)$	$6G$	7680	864
		$f_3=3$	$(1,1,1,1,1,1,1)$	$(0,0,1,0,0,1,0)$	$6H$	23040	288
		$f_4=3$	$(1,1,0,1,1,1,1)$	$(0,0,1,0,0,0,1)$	$6I$	23040	288
$4A$	16	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4S$	4320	1536
		$f_2=1$	$(1,0,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4T$	4320	1536
		$f_3=3$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4U$	12960	512
		$f_4=3$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4V$	12960	512
		$f_5=4$	$(1,1,1,1,1,0,1)$	$(0,0,1,1,1,0,0)$	$8A$	17280	384
		$f_6=4$	$(1,1,1,1,1,1,0)$	$(0,0,1,1,1,0,0)$	$8B$	17280	384
$4B$	8	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4W$	8640	768
		$f_2=1$	$(0,1,1,1,0,0,1)$	$(0,0,0,0,0,0,0)$	$4X$	8640	768
		$f_3=3$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4Y$	25920	256
		$f_4=3$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$4Z$	25920	256
$4C$	16	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AA$	12960	512
		$f_2=1$	$(1,1,1,1,1,0,0)$	$(0,0,0,0,0,0,0)$	$4AB$	12960	512
		$f_3=1$	$(1,1,1,0,1,1,0)$	$(0,0,0,0,0,0,0)$	$4AC$	12960	512
		$f_4=1$	$(1,1,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AD$	12960	512
		$f_5=2$	$(1,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$4AE$	25920	256
		$f_6=2$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4AF$	25920	256
		$f_7=4$	$(1,1,1,1,1,0,1)$	$(1,0,0,0,1,0,0)$	$8C$	51840	128
		$f_8=4$	$(1,1,1,1,1,1,0)$	$(1,0,0,0,1,0,0)$	$8D$	51840	128

Table 3(continued)

$[g]_{O_6^-(2)}$	k	f_j	d_j	w	$[x]_{2^7:O_6^-(2)}$	$\lvert[x]_{2^7:O_6^-(2)}\rvert$	$\lvert C_{2^7:O_6^-(2)}(x)\rvert$
$4D$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AG$	51840	128
		$f_2 = 1$	$(1,1,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AH$	51840	128
		$f_3 = 1$	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AI$	51840	128
		$f_4 = 1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AJ$	51840	128
		$f_5 = 1$	$(1,1,1,1,0,1,1)$	$(0,1,1,0,0,0,1)$	$8E$	51840	128
		$f_6 = 1$	$(1,1,1,0,1,1,1)$	$(0,1,1,0,0,0,1)$	$8F$	51840	128
		$f_7 = 1$	$(1,1,1,1,1,1,1)$	$(0,1,1,0,0,0,1)$	$8G$	51840	128
		$f_8 = 1$	$(1,1,0,1,1,1,1)$	$(0,1,1,0,0,0,1)$	$8H$	51840	128
$5A$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$5A$	82944	80
		$f_2 = 1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,1,1)$	$10A$	82944	80
		$f_3 = 1$	$(0,1,1,1,1,1,1)$	$(0,1,1,1,1,1,0)$	$10B$	82944	80
		$f_4 = 1$	$(1,0,1,1,1,1,1)$	$(0,1,1,1,1,0,1)$	$10C$	82944	80
		$f_5 = 2$	$(1,1,1,0,1,1,1)$	$(1,1,1,0,1,1,1)$	$10D$	165888	40
		$f_6 = 2$	$(1,1,1,1,0,1,1)$	$(1,1,1,0,1,0,0)$	$10E$	165888	40
$6A$	2	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6J$	46080	144
		$f_2 = 1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$6K$	46080	144
$6B$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6L$	23040	288
		$f_2 = 1$	$(0,0,1,0,0,0,1)$	$(0,0,0,0,0,0,0)$	$6M$	23040	288
		$f_3 = 3$	$(1,1,0,0,0,1,0)$	$(1,1,1,0,1,0,0)$	$12A$	69120	96
		$f_4 = 3$	$(1,0,1,1,1,1,1)$	$(0,0,0,1,0,1,0)$	$12B$	69120	96
$6C$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6N$	46080	144
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,1,1,1,0,1)$	$12C$	46080	144
		$f_3 = 1$	$(1,1,0,0,0,0,1)$	$(0,0,1,1,1,0,1)$	$12D$	46080	144
		$f_4 = 1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$6O$	46080	144
$6D$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6P$	23040	288
		$f_2 = 1$	$(0,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$6Q$	23040	288
		$f_3 = 3$	$(0,0,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$6R$	69120	96
		$f_4 = 3$	$(1,1,0,0,0,0,1)$	$(0,0,0,0,0,0,0)$	$6S$	69120	96
$6E$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6T$	11520	576
		$f_2 = 1$	$(0,1,1,1,0,0,1)$	$(0,0,0,0,0,0,0)$	$6U$	11520	576
		$f_3 = 1$	$(1,1,1,0,1,1,1)$	$(1,0,0,0,0,0,0)$	$12E$	11520	576
		$f_4 = 1$	$(1,1,1,0,1,0,0)$	$(1,0,0,0,0,0,0)$	$12F$	11520	576
		$f_5 = 3$	$(0,1,0,0,0,1,0)$	$(1,0,0,0,0,0,0)$	$12G$	34560	192
		$f_6 = 3$	$(0,1,0,0,0,0,1)$	$(1,0,0,0,0,0,0)$	$12H$	34560	192
		$f_7 = 3$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$6V$	34560	192
		$f_8 = 3$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$6W$	34560	192
$6F$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6X$	34560	192
		$f_2 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$6Y$	34560	192
		$f_3 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,1,1,1,0,0)$	$12I$	34560	192
		$f_4 = 1$	$(1,1,0,1,1,1,1)$	$(0,0,1,1,1,0,0)$	$12J$	34560	192
		$f_5 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,1,0,0,1,1)$	$12K$	69120	96
		$f_6 = 2$	$(1,1,1,1,1,0,1)$	$(0,0,1,0,0,1,1)$	$12L$	69120	96
$6G$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6Z$	138240	48
		$f_2 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,1,0,0,1,0)$	$12M$	138240	48
		$f_3 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$6AA$	138240	48
		$f_4 = 1$	$(1,1,1,1,0,1,1)$	$(0,0,1,0,0,1,0)$	$12N$	138240	48
$8A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$8I$	207360	32
		$f_2 = 1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$8J$	207360	32
		$f_3 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$8K$	207360	32
		$f_4 = 1$	$(1,0,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$8L$	207360	32
$9A$	2	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$9A$	368640	18
		$f_2 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,1,1)$	$18A$	368640	18

Table 3(continued)

$[g]_{O_6^-(2)}$	k	f_j	d_j	w	$[x]_{2^7:O_6^-(2)}$	$\lvert[x]_{2^7:O_6^-(2)}\rvert$	$\lvert C_{2^7:O_6^-(2)}(x)\rvert$
$10A$	4	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$10F$	165888	40
		$f_2=1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$10G$	165888	40
		$f_3=1$	$(1,1,1,0,1,1,1)$	$(0,1,1,1,1,1,0)$	$20A$	165888	40
		$f_4=1$	$(1,1,1,1,0,1,1)$	$(0,1,1,1,1,1,0)$	$20B$	165888	40
$12A$	2	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$12O$	276480	24
		$f_2=1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$12P$	276480	24
$12B$	4	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$12Q$	138240	48
		$f_2=1$	$(1,1,1,1,1,1,0)$	$(0,0,1,1,1,0,0)$	$24A$	138240	48
		$f_3=1$	$(1,1,1,1,1,0,1)$	$(0,0,1,1,1,0,0)$	$24B$	138240	48
		$f_4=1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$12R$	138240	48

4. The Inertia Groups of $2^7{:}O_6^-(2)$

The action of $O_6^-(2)$ on the conjugacy classes of 2^7 determine 6 orbits being formed, with respective lengths of $1, 1, 27, 27, 36$ and 36. Therefore by Brauer's Theorem [7], $O_6^-(2)$ acting on $Irr(2^7)$ will also form 6 orbits of lenghts 1, r, s, t, u and v such that $r + s + t + u + v = 127$. Hence there are six inertia groups $\bar{H}_i = 2^7{:}H_i$, $i =$ 1,2,3,4,5 and 6, such that $[G{:}H_1]$=1, $[G{:}H_2] = r$, $[G{:}H_3]$= s, $[G{:}H_4] = t$, $[G{:}H_5]$= u, and $[G{:}H_6] = v$. The H_i's are the factor groups of the inertia groups, which are maximal subgroups or sit in the maximal subgroups of $O_6^-(2)$.

The sum of the number of conjugacy classes of these inertia factors must be in total equal to 130, that is, the number of the conjugacy classes of $2^7{:}O_6^-(2)$. Since $O_6^-(2) \cong U_4(2){:}2$, then by checking all the indices of maximal subgroups of $O_6^-(2)$ in the ATLAS [5] and all combinations that can possibly satisfy the previous two facts, we deduce that $s = t = 27$, $u = v = 36$ and r= 1 are the only lengths of these orbits. $2^4{:}S_5$ and $S_6 \times 2$ are the only copies of maximal subgroups of $O_6^-(2)$, up to isomorphism, that have indices of 27 and 36 in $O_6^-(2)$. Hence $H_1 = H_2 = O_6^-(2)$, $H_3 = H_4 = 2^4{:}S_5$, $H_5 = H_6 = S_6 \times 2$. The inertia groups $2^4{:}S_5$ and $S_6 \times 2$ are constructed from elements within $O_6^-(2)$ and the generators are as follows :

- $S_6 \times 2 = \langle \alpha_1, \alpha_2, \alpha_3 \rangle$, $\alpha_1 \in 4A$,$\alpha_2 \in 6E$, $\alpha_3 \in 6F$ where

$$\alpha_1 = \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&1&0\\ 0&0&1&1&1&1&0\\ 1&1&0&1&1&0&0\\ 1&0&1&1&1&1&1\\ 1&0&1&1&1&0&0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0\\ 0&1&0&0&0&0&1\\ 0&0&1&0&0&0&1\\ 0&0&0&0&1&0&0\\ 0&0&1&1&1&1&1\\ 0&0&1&1&1&0&0 \end{pmatrix}, \quad \alpha_3 = \begin{pmatrix} 1&0&0&0&0&0&0\\ 1&1&1&1&0&0&1\\ 1&0&1&0&0&0&0\\ 1&1&1&0&0&1&0\\ 1&1&0&0&0&1&1\\ 1&0&1&1&1&1&1\\ 1&0&1&1&1&0&0 \end{pmatrix}$$

- $2^4{:}S_5 = \langle \lambda_1, \lambda_2, \lambda_3 \rangle$, $\lambda_1 \in 2C$,$\lambda_2 \in 4C$, $\lambda_3 \in 4D$ where

$$\lambda_1 = \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 1&1&1&1&0&0&0\\ 0&0&0&1&0&0&0\\ 1&1&0&1&1&0&0\\ 0&0&0&1&0&0&1\\ 0&0&0&1&0&1&0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0&0&1&0&1&1&0\\ 0&1&0&0&0&0&0\\ 1&1&1&0&0&1&1\\ 1&0&1&1&0&1&1\\ 1&1&0&0&0&1&0\\ 1&0&1&0&0&0&1\\ 1&0&1&0&0&1&0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 0&1&1&0&1&0&1\\ 0&1&0&0&0&0&0\\ 1&1&0&0&1&0&1\\ 1&0&1&1&0&1&1\\ 0&1&1&1&1&1&1\\ 0&0&1&1&0&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}$$

5. The Fusions of $2^4{:}S_5$ and $S_6 \times 2$ into $O_6^-(2)$

We obtain the fusion maps of the inertia factors $2^4{:}S_5$ and $S_6 \times 2$ into $O_6^-(2)$, by using direct matrix conjugation in $O_6^-(2)$ and the permutation characters of $O_6^-(2)$ on $2^4{:}S_5$ and $S_6 \times 2$. MAGMA[3] was used for the various computations. The fusion maps of the inertia factor groups into $O_6^-(2)$ are shown in the Table 4 and Table 5.

Table 4. The fusion of $2^4{:}S_5$ into $O_6^-(2)$

$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{O_6^-(2)}$	$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{O_6^-(2)}$	$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{O_6^-(2)}$	$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{O_6^-(2)}$
$1A$	$1A$	$2E$	$2C$	$4D$	$4D$	$6C$	$6E$
$2A$	$2B$	$3A$	$3B$	$4E$	$4C$	$8A$	$8A$
$2B$	$2C$	$4A$	$4A$	$5A$	$5A$	$12A$	$12B$
$2C$	$2A$	$4B$	$4C$	$6A$	$6F$		
$2D$	$2D$	$4C$	$4B$	$6B$	$6B$		

Table 5. The fusion of $S_6 \times 2$ into $O_6^-(2)$

$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{O_6^-(2)}$	$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{O_6^-(2)}$	$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{O_6^-(2)}$	$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{O_6^-(2)}$
$1A$	$1A$	$2F$	$2D$	$4C$	$4D$	$6D$	$6F$
$2A$	$2A$	$2G$	$2C$	$4D$	$4D$	$6E$	$6G$
$2B$	$2C$	$3A$	$3B$	$5A$	$5A$	$6F$	$6E$
$2C$	$2D$	$3B$	$3C$	$6A$	$6C$	$10A$	$10A$
$2D$	$2A$	$4A$	$4C$	$6B$	$6E$		
$2E$	$2B$	$4B$	$4A$	$6C$	$6D$		

6. The Fischer-Clifford Matrices of $2^7{:}O_6^-(2)$

Having obtained the fusions of the inertia factor groups $2^4{:}S_5$ and $S_6 \times 2$ into $O_6^-(2)$, we are now able to compute the Fischer-Clifford matrices of the group $2^7{:}O_6^-(2)$. We will use the properties discussed in Section 5.2.2 (see Mpono [13]) to help us in the construction of these matrices. Note that all the relations hold since 2^7 is an elementary abelian group. The reader is encouraged to consult [1], [2], [6], [11], [12], [13], [16] and [17] for full details around the theory and computation of Fischer-Clifford matrices.

The following additional information are needed sometimes to compute these entries:

(1) For χ a character of any group H and $h \in H$, we have $|\chi(h)| \leq \chi(1_H)$, where 1_H is the identity element of H.

(2) For χ a character of any group H and h a $p-$singular element of H, where p is a prime, then we have $\chi(h) \equiv \chi(h^p) \text{mod} p$.

(3) For any irreducible character χ of a group H and for $h_i \in C_i$ then $d_i = \frac{b_i \chi(h_i)}{\chi(1_H)}$ is an algebraic integer, where C_i is the ith conjugacy class of H and $b_i = |C_i| = [H{:}C_H(h_i)]$. It is clear if $d_i \in \mathbb{Q}$, then $d_i \in \mathbb{Z}$.

For each class representative $g \in O_6^-(2)$, we construct a Fischer-Clifford matrix $M(g)$ which are listed in Table 6 .

TABLE 6. The Fischer-Clifford Matrices of $2^7{:}O_6^-(2)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 27 & -27 & 5 & -5 & 3 & -3 \\ 27 & 27 & -5 & -5 & 3 & 3 \\ 36 & -36 & -4 & 4 & -4 & 4 \\ 36 & 36 & 4 & 4 & -4 & -4 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 15 & -15 & -5 & 5 & -3 & 3 & 1 & -1 \\ 15 & 15 & -5 & -5 & 3 & 3 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 15 & -15 & 5 & -5 & 3 & -3 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 15 & 15 & 5 & 5 & -3 & -3 & -1 & -1 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 3 & -3 & 3 & -3 & -1 & 1 \\ 3 & 3 & 3 & 3 & -1 & -1 \\ 12 & -12 & -4 & 4 & 0 & 0 \\ 12 & 12 & -4 & -4 & 0 & 0 \end{pmatrix}$	$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 6 & -6 & 6 & -6 & 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 6 & 6 & -6 & -6 & 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & -2 & 2 & -2 & -2 & 2 & 2 & -2 & 0 & 0 \\ 6 & -6 & -6 & 6 & -2 & 2 & -2 & 2 & 0 & 0 \\ 2 & 2 & -2 & -2 & -2 & -2 & 2 & 2 & 0 & 0 \\ 6 & 6 & 6 & 6 & -2 & -2 & -2 & -2 & 0 & 0 \end{pmatrix}$
$M(2D) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\ 3 & -3 & 3 & -3 & -1 & -1 & 1 & 1 \\ 3 & 3 & 3 & 3 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 3 & -3 & -3 & 3 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\ 3 & 3 & -3 & -3 & -1 & 1 & 1 & -1 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 9 & -9 & -3 & 3 & -1 & 1 \\ 9 & 9 & -3 & -3 & 1 & 1 \\ 6 & -6 & 2 & -2 & 2 & -2 \\ 6 & 6 & 2 & 2 & -2 & -2 \end{pmatrix}$	$M(3C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$

Table 6 (continued)

$M(g)$	$M(g)$
$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 6 & -6 & -2 & 2 & 0 & 0 \\ 6 & 6 & -2 & -2 & 0 & 0 \end{pmatrix}$	$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$
$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 4 & -4 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 4 & 4 & -4 & -4 & 0 & 0 & 0 & 0 \\ 2 & -2 & -2 & 2 & -2 & 2 & 0 & 0 \\ 2 & 2 & 2 & 2 & -2 & -2 & 0 & 0 \end{pmatrix}$	$M(4D) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{pmatrix}$
$M(5A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \end{pmatrix}$	$M(6A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$	$M(6C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$
$M(6D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$	$M(6E) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 3 & -3 & 3 & -3 & 1 & -1 & 1 & -1 \\ 3 & 3 & -3 & -3 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 3 & -3 & -3 & 3 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 3 & 3 & 3 & 3 & -1 & -1 & -1 & -1 \end{pmatrix}$
$M(6F) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 0 & 0 \end{pmatrix}$	$M(6G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$	$M(9A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(10A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(12B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$	

7. Character Table of $2^7{:}O_6^-(2)$

The character table of $2^7{:}O_6^-(2)$ was constructed through the above Fischer-Clifford matrices and the character tables of the 6 inertia factors groups together with their respective fusion maps into $O_6^-(2)$. We used MAGMA [3] to compute the character tables of the inertia factor groups. We partitioned the set of irreducible characters of $2^7{:}O_6^-(2)$ into 6 blocks B_1, B_2, B_3, B_4, B_5 and B_6 corresponding to the inertia factor groups H_1, H_2, H_3, H_4, H_5 and H_6 respectively. Therefore $Irr(2^7{:}O_6^-(2)) = \bigcup_{i=1}^{6} B_i$, where $B_1 = \{\chi_j | 1 \leq j \leq 25\}$, $B_2 = \{\chi_j | 26 \leq j \leq 50\}$, $B_3 = \{\chi_j | 51 \leq j \leq 68\}$, $B_4 = \{\chi_j | 69 \leq j \leq 86\}$, $B_5 = \{\chi_j | 87 \leq j \leq 108\}$ and $B_6 = \{\chi_j | 109 \leq j \leq 130\}$. For a thorough discussion around the construction of a character table of a finite extension group of the type of $2^7{:}O_6^-(2)$, through the use of its Fischer-Clifford matrices, the reader is referred to [1], [2], [11], [12], [13] and [16]. The character table of $2^7{:}O_6^-(2)$ can be found in [14] or the reader can obtain it directly from the authors. The consistency and accuracy of the character table of $2^7{:}O_6^-(2)$ have been tested by using Programme C [14] written in GAP [15].

8. The Fusion of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$

We use the results of Section 3 to compute the power maps of the elements of $2^7{:}O_6^-(2)$ which are listed in Table 7.

Table 7. The Power Maps of the Elements of $2^7{:}O_6^-(2)$

$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$	2	3	5	7	$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$	2	3	5	7
$1A$	$1A$					$2A$	$2F$	$1A$			
	$2A$	$1A$					$2G$	$1A$			
	$2B$	$1A$					$4A$	$2D$			
	$2C$	$1A$					$4B$	$2D$			
	$2D$	$1A$					$4C$	$2D$			
	$2E$	$1A$					$4D$	$2D$			
							$2H$	$1A$			
							$2I$	$1A$			
$2B$	$2J$	$1A$				$2C$	$2N$	$1A$			
	$2K$	$1A$					$2O$	$1A$			
	$2L$	$1A$					$4G$	$2C$			
	$2M$	$1A$					$4H$	$2C$			
	$4E$	$2C$					$4I$	$2C$			
	$4F$	$2C$					$4J$	$2C$			
							$2P$	$1A$			
							$2Q$	$1A$			
							$4K$	$2D$			
							$4L$	$2D$			
$2D$	$2R$	$1A$				$3A$	$3A$		$1A$		
	$2S$	$1A$					$6A$	$3A$	$2A$		
	$4M$	$2D$									
	$4N$	$2D$									
	$4O$	$2C$									
	$4P$	$2D$									
	$4Q$	$2D$									
	$4R$	$2C$									

Table 7(continued)

$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$	2	3	5	7	$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$	2	3	5	7
$3B$	$3B$		$1A$			$3C$	$3C$		$1A$		
	$6B$	$3B$	$2A$				$6G$	$3C$	$2A$		
	$6C$	$3B$	$2D$				$6H$	$3C$	$2D$		
	$6D$	$3B$	$2E$				$6I$	$3C$	$2E$		
	$6E$	$3B$	$2B$								
	$6F$	$3B$	$2C$								
$4A$	$4S$	$2N$				$4B$	$4W$	$2J$			
	$4T$	$2N$					$4X$	$2J$			
	$4U$	$2N$					$4Y$	$2L$			
	$4V$	$2N$					$4Z$	$2L$			
	$8A$	$4H$									
	$8B$	$4H$									
$4C$	$4AA$	$2N$				$4D$	$4AG$	$2N$			
	$4AB$	$2N$					$4AH$	$2N$			
	$4AC$	$2N$					$4AI$	$2P$			
	$4AD$	$2N$					$4AJ$	$2P$			
	$4AE$	$2N$					$8E$	$4I$			
	$4AF$	$2N$					$8F$	$4I$			
	$8C$	$4I$					$8G$	$4H$			
	$8D$	$4I$					$8H$	$4H$			
$5A$	$5A$			$1A$		$6A$	$6J$	$3A$	$2J$		
	10A	$5A$		2A			$6K$	$3A$	$2K$		
	$10B$	$5A$		$2D$							
	$10C$	$5A$		$2E$							
	$10D$	$5A$		$2B$							
	$10E$	$5A$		2C							
$6B$	$6L$	$3B$	$2J$			$6C$	$6N$	$3C$	$2F$		
	$6M$	$3B$	$2K$				$12C$	$6H$	$4D$		
	$12A$	$6F$	$4E$				$12D$	$6H$	$4C$		
	$12B$	$6F$	$4F$				$6O$	$3C$	$2G$		
$6D$	$6P$	$3C$	$2J$			$6E$	$6T$	$3B$	$2F$		
	$6Q$	$3C$	$2K$				$6U$	$3B$	$2G$		
	$6R$	$3C$	$2L$				$12E$	$6C$	$4C$		
	$6S$	$3C$	$2M$				$12F$	$6C$	$4D$		
							$12G$	$6C$	$4A$		
							$12H$	$6C$	$4B$		
							$6V$	$3B$	$2H$		
							$6W$	$3B$	$2I$		
$6F$	$6X$	$3B$	$2N$			$6G$	$6Z$	$3C$	$2R$		
	$6Y$	$3B$	$2O$				$12M$	$6H$	$4N$		
	$12I$	$6F$	$4H$				$6AA$	$3C$	$2S$		
	$12J$	$6F$	$4G$				$12N$	$6H$	$4M$		
	$12K$	$6C$	$4K$								
	$12L$	$6C$	$4L$								
$8A$	$8I$	$4W$				$9A$	$9A$		$3A$		
	$8J$	$4W$					$18A$		$6A$		
	$8K$	$4Y$									
	$8L$	$4Y$									
$10A$	$10F$	$5A$		$2F$		$12A$	$12O$	$6J$	$4W$		
	$10G$	$5A$		$2G$			$12P$	$6J$	$4X$		
	$20A$	$10B$		$4B$							
	$20B$	$10B$		$4A$							
$12B$	$12Q$	$6X$	$4S$								
	$24A$	$12I$	$8A$								
	$24B$	$12I$	$8B$								
	$12R$	$6X$	$4T$								

Let $\chi(2^7{:}Sp_6(2)|2^7{:}O_6^-(2)$ be the permutation character of degree 28 of $2^7{:}Sp_6(2)$ acting on $2^7{:}O_6^-(2)$. We obtain that $\chi(2^7{:}Sp_6(2)|2^7{:}O_6^-(2)) = 1a+27a$. We are able to obtain the partial fusion of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$ by using the information provided by the conjugacy classes of the elements of $2^7{:}O_6^-(2)$ and $2^7{:}Sp_6(2)$, their power maps, together with the permutation character of $2^7{:}Sp_6(2)$ of degree 28 and the fusion map of $O_6^-(2)$ into $Sp_6(2)$. We used the technique of set intersections for characters to restrict $63a$, $63b$, $36a$, $36b$, $28a$, $28b \in Irr(2^7{:}Sp_6(2))$ to $2^7{:}O_6^-(2)$ to determine fully the fusion of the classes of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$. We refer the reader for detailed information regarding the above set intersections technique to Ali [1], Moori and Ali [2], Moori [11], Moori and Mpono [12] and Mpono [13].

Let ζ be the character afforded by the regular representation of $O_6^-(2)$. We obtain that $\zeta = \Sigma_{i=1}^{25}\alpha_i\Phi_i$, where $\Phi_i \in Irr(O_6^-(2))$ and $\alpha_i = deg(\Phi_i)$. Then ζ can be regarded as a character of $2^7{:}O_6^-(2)$ which contains 2^7 in its kernel such that

$$\zeta(x) = \begin{cases} |O_6^-(2)| & \text{if } x \in 2^7 \\ 0 & \text{otherwise} . \end{cases}$$

If ϕ is a character of $2^7{:}Sp_6(2)$ than we have that

$$\begin{aligned} <\zeta,\phi>_{2^7:O_6^-(2)} = & \frac{1}{|2^7:O_6^-(2)|}\{\zeta(1A)\phi(1A)+\zeta(2A)\phi(2A)+27\zeta(2B)\phi(2B)+27\zeta(2C)\phi(2C)+ \\ & 36\zeta(2D)\phi(2D)+36\zeta(2E)\phi(2E)\} \\ = & \frac{1}{|2^7:O_6^-(2)|}\{|O_6^-(2)|(\phi(1A)+\phi(2A)+27\phi(2B)+27\phi(2C)+36\phi(2D)+36\phi(2E))\} \\ = & \frac{1}{128}\{\phi(1A)+\phi(2A)+27\phi(2B)+27\phi(2C)+36\phi(2D)+36\phi(2E)\} \\ = & <\phi_{2^7},1_{2^7}> . \end{aligned}$$

Here 1_{2^7} is the identity character of 2^7 and ϕ_{2^7} is the restriction of ϕ to 2^7. We obtain that

$$\phi_{2^7} = a_1\theta_1 + a_2\theta_2 + a_3\theta_3 + a_4\theta_4 + a_5\theta_5 + a_6\theta_6,$$

where $a_i \in \mathbb{N}\cup\{0\}$ and θ_i are the sums of the irreducible characters of 2^7 which are in the same orbit under the action of $O_6^-(2)$ on $Irr(2^7)$, for $i \in \{1,2,3,4,5,6\}$. Let $\varphi_j \in Irr(2^7)$, where $j \in \{1,2,3,...,130\}$. Then we obtain that

$$\theta_1 = \varphi_1 \ , \quad deg(\theta_1) = \ 1$$
$$\theta_2 = \varphi_2 \ , \quad deg(\theta_2) = \ 1$$
$$\theta_3 = \sum_{j=3}^{29} \varphi_j \ , \quad deg(\theta_3) = \ 27$$
$$\theta_4 = \sum_{j=30}^{56} \varphi_j \ , \quad deg(\theta_4) = \ 27$$
$$\theta_5 = \sum_{j=57}^{92} \varphi_j \ , \quad deg(\theta_5) = \ 36$$
$$\theta_6 = \sum_{j=93}^{128} \varphi_j \ , \quad deg(\theta_6) = \ 36 \ .$$

Hence

$$\phi_{2^7} = a_1\varphi_1 + a_2\varphi_2 + a_3\sum_{j=3}^{29}\varphi_j + a_4\sum_{j=30}^{56}\varphi_j + a_5\sum_{j=57}^{92}\varphi_j + a_6\sum_{j=93}^{128}\varphi_j,$$

and therefore

$$\begin{aligned}\langle \phi_{2^7}, \phi_{2^7}\rangle = & \quad a_1^2 + a_2^2 + 27a_3^2 + 27a_4^2 + 36a_5^2 + 36a_6^2 \\ = & \quad \frac{1}{128}\{\phi(1A)\phi(1A) + \phi(2A)\phi(2A) + 27\phi(2B)\phi(2B) + 27\phi(2C)\phi(2C) + \\ & \quad 36\phi(2D)\phi(2D) + 36\phi(2E)\phi(2E)\} \ ,\end{aligned}$$

where $a_1 = <\zeta, \phi>_{2^7{:}O_6^-(2)}$.

We apply the above results to some of the irreducible characters of $2^7{:}Sp_6(2)$, which in this case are $\phi_1 = 28a$, $\phi_2 = 28b$, $\phi_3 = 36a$, $\phi_4 = 36b$, $\phi_5 = 63a$ and $\phi_6 = 63b$. Their respective degrees are 28, 28, 36, 36, 63 and 63. For ϕ_1 we calculate that

$$<\zeta, \phi_1>_{2^7{:}O_6^-(2)} = \frac{1}{128}\{28 + (-28) + 27(-4) + 27(4) + 36(-4) + 36(4)\} = 0 \ .$$

Now $a_1 + a_2 + 27a_3 + 27a_4 + 36a_5 + 36a_6 = 28$, since $deg\phi_1 = 28$. Since $a_1 = 0$, we must have either $a_2 = 1$, $a_4 = a_5 = a_6 = 0$ and $a_3 = 1$ or $a_2 = 1$, $a_3 = a_5 = a_6 = 0$ and $a_4 = 1$. Note that $2^7{:}O_6^-(2)$ does not have irreducible characters of degree 28. We obtain that $(\phi_1)_{2^7{:}O_6^-(2)} = \chi_{26} + \chi_{51}$ if the partial fusion of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$ is taken into consideration. Similarly for ϕ_2, ϕ_3, ϕ_4, ϕ_5 and ϕ_6 we obtain that

$$(\phi_2)_{2^7:O_6^-(2)} = \chi_{27} + \chi_{52}$$
$$(\phi_3)_{2^7:O_6^-(2)} = \chi_{87}$$
$$(\phi_4)_{2^7:O_6^-(2)} = \chi_{90}$$
$$(\phi_5)_{2^7:O_6^-(2)} = \chi_{69} + \chi_{109}$$
$$(\phi_6)_{2^7:O_6^-(2)} = \chi_{70} + \chi_{110}.$$

By making use of the values of ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5 and ϕ_6 on the classes of $2^7{:}Sp_6(2)$ and the values of $(\phi_1)_{2^7:O_6^-(2)}$, $(\phi_2)_{2^7:O_6^-(2)}$, $(\phi_3)_{2^7:O_6^-(2)}$, $(\phi_4)_{2^7:O_6^-(2)}$, $(\phi_5)_{2^7:O_6^-(2)}$ and $(\phi_6)_{2^7:O_6^-(2)}$ on the classes of $2^7{:}O_6^-(2)$ together with the partial fusion, the complete fusion map of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$ is given in the Table 8.

TABLE 8. Fusion of $2^7{:}O_6^-(2)$ into $2^7{:}Sp_6(2)$

$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$ $\longrightarrow$	$[y]_{2^7:Sp_6(2)}$	$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$ $\longrightarrow$	$[y]_{2^7:Sp_6(2)}$
$1A$	$1A$	$1A$	$2A$	$2F$	$2D$
	$2A$	$2A$		$2G$	$2E$
	$2B$	$2C$		$4A$	$4B$
	$2C$	$2B$		$4B$	$4A$
	$2D$	$2C$		$4C$	$4B$
	$2E$	$2B$		$4D$	$4A$
				$2H$	$2F$
				$2I$	$2F$
$2B$	$2J$	$2G$	$2C$	$2N$	$2K$
	$2K$	$2H$		$2O$	$2L$
	$2L$	$2I$		$4G$	$4D$
	$2M$	$2J$		$4H$	$4E$
	$4E$	$4C$		$4I$	$4D$
	$4F$	$4C$		$4J$	$4E$
				$2P$	$2M$
				$2Q$	$2M$
				$4K$	$4F$
				$4L$	$4F$
$2D$	$2R$	$2N$	$3A$	$3A$	$3B$
	$2S$	$2O$		$6A$	$6D$
	$4M$	$4H$			
	$4N$	$4G$			
	$4O$	$4I$			
	$4P$	$4J$			
	$4Q$	$4J$			
	$4R$	$4I$			
$3B$	$3B$	$3A$	$3C$	$3C$	$3C$
	$6B$	$6A$		$6G$	$6E$
	$6C$	$6C$		$6H$	$6G$
	$6D$	$6B$		$6I$	$6F$
	$6E$	$6C$			
	$6F$	$6B$			
$4A$	$4S$	$4R$	$4B$	$4W$	$4K$
	$4T$	$4S$		$4X$	$4L$
	$4U$	$4U$		$4Y$	$4M$
	$4V$	$4T$		$4Z$	$4M$
	$8A$	$8B$			
	$8B$	$8B$			

Table 8 (continued)

$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$ $\longrightarrow$	$[y]_{2^7:Sp_6(2)}$	$[g]_{O_6^-(2)}$	$[x]_{2^7:O_6^-(2)}$ $\longrightarrow$	$[y]_{2^7:Sp_6(2)}$
$4C$	$4AA$	$4N$	$4D$	$4AG$	$4Z$
	$4AB$	$4O$		$4AH$	$4AA$
	$4AC$	$4Q$		$4AI$	$4AC$
	$4AD$	$4P$		$4AJ$	$4AB$
	$4AE$	$4Q$		$8E$	$8C$
	$4AF$	$4P$		$8F$	$8C$
	$8C$	$8A$		$8G$	$8D$
	$8D$	$8A$		$8H$	$8D$
$5A$	$5A$	$5A$	$6A$	$6J$	$6M$
	$10A$	$10A$		$6K$	$6N$
	$10B$	$10B$			
	$10C$	$10C$			
	$10D$	$10B$			
	$10E$	$10C$			
$6B$	$6L$	$6K$	$6C$	$6N$	$6U$
	$6M$	$6L$		$12C$	$12H$
	$12A$	$12C$		$12D$	$12G$
	$12B$	$12C$		$6O$	$6V$
$6D$	$6P$	$6Q$	$6E$	$6T$	$6H$
	$6Q$	$6R$		$6U$	$6I$
	$6R$	$6T$		$12E$	$12B$
	$6S$	$6S$		$12F$	$12A$
				$12G$	$12B$
				$12H$	$12A$
				$6V$	$6J$
				$6W$	$6J$
$6F$	$6X$	$6O$	$6G$	$6Z$	$6W$
	$6Y$	$6P$		$12M$	$12I$
	$12I$	$12E$		$6AA$	$6X$
	$12J$	$12D$		$12N$	$12J$
	$12K$	$12F$			
	$12L$	$12F$			
$8A$	$8I$	$8H$	$9A$	$9A$	$9A$
	$8J$	$8I$		$18A$	$18A$
	$8K$	$8J$			
	$8L$	$8J$			
$10A$	$10F$	$10D$	$12A$	$12O$	$12O$
	$10G$	$10E$		$12P$	$12P$
	$20A$	$20A$			
	$20B$	$20B$			
$12B$	$12Q$	$12M$			
	$24A$	$24B$			
	$24B$	$24B$			
	$12R$	$12N$			

Acknowledgements

The first author would like to thank his supervisor (second author) for his advise and support. The financial support from the NRF and University of the Western Cape are also acknowledged. I am most grateful to my Lord Jesus Christ.

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