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ON THE ADDITIVE GROUP OF A FINITE SKEW BRACE WHOSE MULTIPLICATIVE GROUP IS A PRODUCT OF TWO NON-ABELIAN SIMPLE GROUPS

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Abstract. We study finite skew braces whose multiplicative group is the direct product of two finite non-abelian simple groups. More precisely, we show that if B is a finite skew brace with $(B, \cdot) \cong S \times T$, where S and T are finite non-abelian simple groups, then the additive group $(B, +)$ cannot be supersolvable. This result contributes to the study of the restrictions imposed by the multiplicative group on the additive group of a skew brace, a central question related to the Byott–Vendramin conjecture. The proof combines structural properties of supersolvable groups, the existence of characteristic Hall subgroups, and classification results on finite simple groups admitting subgroups of 2-power index. These tools reduce the possible multiplicative groups to groups of projective linear type and force the occurrence of a section isomorphic to $\mathrm{PSL}_2(7)$. The remaining cases are excluded through an analysis of suitable Sylow subgroups and the induced lambda action.

1. Introduction

Skew braces were introduced by Guarnieri and Vendramin in [9] as a group-theoretic framework for the study of set-theoretic solutions of the Yang–Baxter equation. A *skew brace* is a triple $(B, +, \cdot)$ such that $(B, +)$ and $(B, \cdot)$ are groups and

$$a \cdot (b + c) = a \cdot b - a + a \cdot c \qquad \text{for all } a, b, c \in B.$$

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The groups $(B, +)$ and $(B, \cdot)$ are called, respectively, the *additive group* and the *multiplicative group* of the skew brace.

A central problem in the theory is to understand the relation between these two groups. More precisely, one would like to determine which pairs of groups can occur as the additive and multiplicative groups of the same skew brace, and how structural properties of one group impose restrictions on the other. In this direction, Smoktunowicz and Vendramin proved in [16] that if B is finite and $(B, +)$ is nilpotent, then $(B, \cdot)$ is solvable. This result is related to a question of Byott, originally arising in the context of Hopf–Galois theory [2], and later formulated by Vendramin in terms of skew braces [19]: if B is a finite skew brace and $(B, +)$ is solvable, must $(B, \cdot)$ also be solvable? This is often referred to as the Byott–Vendramin conjecture. Several partial results are known. For instance, the conjecture has been verified for various classes of finite skew braces; see, among others, [8, 14, 3, 13, 5, 4, 18]. Byott also studied the case in which the multiplicative group is a non-abelian simple group [1] and later Tsang in [17] studied the case where $(B, \cdot)$ is quasisimple. In this paper we consider the next natural situation, namely skew braces whose multiplicative group is a direct product of two non-abelian simple groups.

Our main result gives a strong obstruction to the supersolvability of the additive group.

Theorem A. *Let B be a finite skew brace such that $(B, \cdot) \cong S \times T$, where S and T are finite non-abelian simple groups. Then $(B, +)$ is not supersolvable.*

The proof combines several ingredients. First, assuming that $(B, +)$ is supersolvable, one obtains a characteristic Hall subgroup of odd order in the additive group, which becomes a 2-complement in the multiplicative group. This forces each simple factor S and T to have a subgroup of index a power of 2. By the classification result used in [8], the factors are then reduced to groups of projective linear type. A further result of Byott implies that the multiplicative group has a section isomorphic to $\mathrm{PSL}_2(7)$. Using a simple lemma on sections of direct products, this section must come from one of the two simple factors. This reduces the possible multiplicative group to

$$\mathrm{PSL}_2(7) \times \mathrm{PSL}_2(p),$$

with $p + 1$ a power of 2. The remaining cases are excluded by analyzing suitable Sylow subgroups of the additive group and the induced lambda action.

The paper is organized as follows. In Section 2 we collect the preliminary results needed in the proof. In Section 3 we prove the main theorem.

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2. Preliminaries

In this section we collect the few facts that will be used in the proof of the main theorem. Let $B = (B, +, \cdot)$ be a skew brace. Recall that, for every $b \in B$, the map

$$\lambda_b : (B, +) \longrightarrow (B, +), \qquad a \mapsto -b + b \cdot a,$$

is an automorphism of the group $(B, +)$. Moreover, the assignment

$$\lambda : (B, \cdot) \longrightarrow \mathrm{Aut}(B, +), \qquad b \mapsto \lambda_b,$$

is a group homomorphism. A subgroup I of $(B, +)$ is called a *left ideal* of B if it is normal in $(B, +)$ and $\lambda_b(I) \leq I$ for every $b \in B$. In particular, every characteristic subgroup of $(B, +)$ is a left ideal. If I is a left ideal, then $(I, \cdot)$ is a subgroup of $(B, \cdot)$. Given any skew brace B, one can replace the additive group $(B, +)$ by its *opposite group* to obtain a new skew brace

$$B^{\mathrm{opp}} = (B, +^{\mathrm{opp}}, \cdot),$$

called the *opposite skew brace* of B (see [12, Proposition 3.1]).

We shall also use the following standard terminology. A *section* of a group G is a quotient M/N, where $N \trianglelefteq M \leq G$.

Lemma 2.1. *Let S, T be finite groups and suppose that $S \times T$ has a section isomorphic to a non-abelian simple group L. Then L is isomorphic to a section of S or to a section of T.*

Proof. Let $M \leq S \times T$ and $N \trianglelefteq M$ be such that $M/N \cong L$. Set

$$K_S = M \cap (S \times 1), \qquad K_T = M \cap (1 \times T).$$

Since $K_S \trianglelefteq M$ and $K_T \trianglelefteq M$ we have that $K_S N/N$ and $K_T N/N$ are normal subgroups of M/N. Since $M/N \cong L$ and L is simple, each of these two normal subgroups is either trivial or equal to M/N. Moreover, the subgroups K_S and K_T commute elementwise, because $K_S \leq S \times 1$ and $K_T \leq 1 \times T$. Hence their images $K_S N/N$ and $K_T N/N$ commute elementwise in M/N. Since L is non-abelian, these two images cannot both be equal to M/N. Assume first that $K_S N/N = M/N$. Then $M = K_S N$. By the second isomorphism theorem, we obtain

$$M/N = K_S N/N \cong K_S/(K_S \cap N).$$

Since $K_S \leq S \times 1 \cong S$, the quotient $K_S/(K_S \cap N)$ is a section of S. Thus L is isomorphic to a section of S. Similarly, if $K_T N/N = M/N$, then $M = K_T N$, and again by the second isomorphism theorem,

$$M/N = K_T N/N \cong K_T/(K_T \cap N).$$

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Since $K_T \leq 1 \times T \cong T$, this shows that L is isomorphic to a section of T. It remains to consider the case $K_S N/N = 1$ and $K_T N/N = 1$. Equivalently, $K_S \leq N$ and $K_T \leq N$. Let $\pi_S : S \times T \to S$ and $\pi_T : S \times T \to T$ be the two canonical projections. We restrict them to M. Then

$$\ker(\pi_S|_M) = M \cap (1 \times T) = K_T \leq N,$$

and

$$\ker(\pi_T|_M) = M \cap (S \times 1) = K_S \leq N.$$

We claim that $\pi_S|_M$ induces an isomorphism

$$M/N \cong \pi_S(M)/\pi_S(N).$$

Indeed, since $\pi_S(N) \trianglelefteq \pi_S(M)$, the map

$$\overline{\pi}_S : M/N \longrightarrow \pi_S(M)/\pi_S(N), \qquad mN \longmapsto \pi_S(m)\pi_S(N),$$

is well defined and surjective. Its kernel is

$$\ker \overline{\pi}_S = \{\, mN \in M/N : \pi_S(m) \in \pi_S(N) \,\}.$$

If $mN \in \ker \overline{\pi}_S$, then there exists $n \in N$ such that $\pi_S(m) = \pi_S(n)$. Hence $mn^{-1} \in \ker(\pi_S|_M) = K_T \leq N$. Since $n \in N$, it follows that $m \in N$. Therefore $\ker \overline{\pi}_S = 1$. Thus

$$M/N \cong \pi_S(M)/\pi_S(N),$$

which is a section of S. Similarly, L is isomorphic to a section of T. Hence, in all cases, L is isomorphic to a section of S or to a section of T, as required. □

Lemma 2.2. *Let G be a finite non-abelian simple group admitting a 2-complement. Then*

$$G \cong \mathrm{PSL}_2(p),$$

where $p \geq 7$ is a prime and $p+1$ is a power of 2.

Proof. By [8, Lemma 2.4], there exist an integer $n \geq 2$ and a prime power q such that

$$G \cong \mathrm{PSL}_n(q) \qquad \text{and} \qquad \frac{q^n-1}{q-1} = 2^t$$

for some $t \in \mathbb{N}_{>0}$. Since

$$\frac{q^n-1}{q-1} = 1 + q + \cdots + q^{n-1}$$

is even, q must be odd. Write $q = p^f$, where p is an odd prime and $f \in \mathbb{N}_{>0}$. Then

$$1 + p^f + \cdots + p^{f(n-1)} = 2^t.$$

Every summand on the left-hand side is odd, and hence n is even. Write $n = 2k$. We obtain

$$2^t = \frac{p^{2fk}-1}{p^f-1} = \left(\frac{p^{fk}-1}{p^f-1}\right)(p^{fk}+1).$$

Consequently, both factors on the right-hand side are powers of 2. In particular,

$$p^{fk} + 1 = 2^s$$

for some $s \in \mathbb{N}_{>0}$. By the Catalan–Mihăilescu theorem (see the main theorem of [15]), we must have $fk = 1$. Thus $f = k = 1$, so $n = 2$, $q = p$, and

$$G \cong \mathrm{PSL}_2(p), \qquad p + 1 = 2^t.$$

Since G is non-abelian simple, necessarily $p \geq 7$. □

The following result will be used to force the occurrence of $\mathrm{PSL}_2(7)$.

Lemma 2.3. *Let B be a finite skew brace such that $(B, +)$ is solvable and $(B, \cdot)$ is not solvable. Then $(B, \cdot)$ has a section isomorphic to* $\mathrm{PSL}_2(7)$.

Proof. This is the consequence of Byott's result used in [3, Corollary 3.3]. □

We shall need the following elementary observation.

Lemma 2.4. *Let $n \in \mathbb{N}_{>0}$. Then $G = \mathrm{PSL}_2(7)^n$ has no subgroup of index 3^m with $1 \leq m \leq n$.*

Proof. Write $G = G_1 \times \cdots \times G_n$ where $G_i \cong \mathrm{PSL}_2(7)$ for each i. Let $H \leq G$ be a subgroup such that $[G : H] = 3^m$. If $G_i \leq H$ for every i, then

$$G = \langle G_1, \ldots, G_n \rangle \leq H,$$

and hence $H = G$, a contradiction. Therefore $G_j \not\leq H$ for some j. It follows that $G_j \cap H < G_j$, so $[G_j : G_j \cap H] > 1$. On the other hand, $[G_j : G_j \cap H] = [G_j H : H]$ so this index divides $[G : H] = 3^m$. Hence $[G_j : G_j \cap H]$ is a non-trivial power of 3. This is impossible, since $\mathrm{PSL}_2(7)$ has no subgroups of index equal to a non-trivial power of 3. Therefore no such subgroup H exists. □

Finally, we record the comparison result between the derived series of the additive and multiplicative groups.

Lemma 2.5. *Let $(B, +, \cdot)$ be a finite skew brace such that $(B, +)$ is solvable, and let H be a characteristic subgroup of $(B, +)$. If $\mathrm{Aut}((B, +)/H)$ is solvable, then there exists $k \in \mathbb{N}_{>0}$ such that*

$$(B, \cdot)^{(k)} \leq H.$$

Proof. Since $(B, +)$ is solvable, there is $n \in \mathbb{N}_{>0}$ such that $(B, +)^{(n)} \leq H$. By [13, Corollary 3.2] there is $m \in \mathbb{N}_{>0}$ such that $(B, \cdot)^{(n+m)} \leq H$. □

Lemma 2.6. *Let G be a finite group and let P be an abelian Sylow p-subgroup of G. If*

$$P \leq G' \cap Z(G),$$

then $P = 1$.

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Proof. By [10, Theorem III.14.3],

$$P \cap G' \cap Z(G) = 1.$$

The final assertion follows immediately. $\square$

3. Proof of Theorem A

Proof of Theorem A. Suppose, for contradiction, that the statement is false, and let B be a counterexample. Write $|B| = 2^{\alpha} m$, where $\alpha \in \mathbb{N}_{\geq 0}$ and $m \in \mathbb{N}_{>0}$ is odd. Since $(B, \cdot) \cong S \times T$ is not solvable, the Feit–Thompson theorem implies that $|B|$ is even. Hence $\alpha > 0$. Since $(B, +)$ is supersolvable, [10, Theorem VI.9.1] ensures the existence of a characteristic subgroup $H \leq (B, +)$ of order m. Hence $(H, \cdot)$ is a 2-complement of $(B, \cdot)$. Write $(B, \cdot) = G_1 \times G_2$, where $G_1 \simeq S$ and $G_2 \simeq T$. Let $i \in \{1, 2\}$. Since each G_i is normal in $(B, \cdot)$, we have $G_i H \leq (B, \cdot)$, and $[G_i : G_i \cap H] = [G_i H : H]$ is a power of 2. If $G_i \leq H$ for some i, then G_i has odd order. By the Feit–Thompson theorem (see the main Theorem of [6]), G_i is solvable, a contradiction. Therefore $G_j \not\leq H$ for every j, and so G_j has a subgroup of index a power of 2. Since $G_j \cap H \leq H$, it follows that $G_j \cap H$ is a 2-complement of G_j. Thus S and T are non-abelian simple groups admitting a 2-complement. By Lemma 2.2, there exist primes $p, r \geq 7$ such that

$$S \cong \mathrm{PSL}_2(p) \qquad \text{and} \qquad T \cong \mathrm{PSL}_2(r),$$

where both $p+1$ and $r+1$ are powers of 2.

By Lemma 2.3, the group $(B, \cdot)$ has a section isomorphic to $\mathrm{PSL}_2(7)$. By Lemma 2.1, we may assume, after interchanging S and T if necessary, that S has a section isomorphic to $\mathrm{PSL}_2(7)$.

Let $K \leq S$ and $N \trianglelefteq K$ be such that

$$K/N \cong \mathrm{PSL}_2(7).$$

Since $\mathrm{PSL}_2(7)$ is not solvable, the group K is not solvable. By the classification of the subgroups of $\mathrm{PSL}_2(p)$; see [10, Theorem II.8.27], it follows that K is simple. Hence $N = 1$, and therefore

$$\mathrm{PSL}_2(7) \leq \mathrm{PSL}_2(p).$$

Applying [10, Theorem 8.27] once again, we conclude that $p = 7$. Thus

$$S \cong \mathrm{PSL}_2(7).$$

We distinguish two cases.

The case r=7. In this case

$$(B, \cdot) \cong \mathrm{PSL}_2(7) \times \mathrm{PSL}_2(7).$$

Since $|\mathrm{PSL}_2(7)| = 2^3 \cdot 3 \cdot 7$, we have $v_3(|B|) = 2$. Assume that

$$3 \mid [(B, +) : (B, +)'].$$

Then by [8, Lemma 2.1] there exists a characteristic subgroup K of $(B,+)$ such that

$$(B,+)/K \cong C_3^m, \qquad m \in \{1,2\}.$$

Hence $(B,\cdot)$ admits a subgroup of index 3^m, which is impossible by Lemma 2.4. Therefore

$$3 \nmid [(B,+):(B,+)'].$$

Thus every Sylow 3-subgroup of $(B,+)$ is contained in $(B,+)'$. By [10, Theorem VI.9.1], the derived subgroup $(B,+)'$ is nilpotent. Hence $(B,+)$ has a normal Sylow 3-subgroup. Let P_3 denote the Sylow 3-subgroup of $(B,+)$. Since P_3 is the unique Sylow 3-subgroup of $(B,+)$, it is characteristic in $(B,+)$. Therefore its centralizer $C_{(B,+)}(P_3)$ is also characteristic in $(B,+)$. Thus, by [11, X.19, Corollary, p. 339],

$$(B,+)/C_{(B,+)}(P_3) \leq \mathrm{Aut}(P_3).$$

We now show that $\mathrm{Aut}((B,+)/C_{(B,+)}(P_3))$ is solvable. If P_3 is cyclic, then

$$\mathrm{Aut}(P_3) \cong \mathrm{Aut}(C_9) \cong C_6,$$

and therefore $(B,+)/C_{(B,+)}(P_3)$ is cyclic and $\mathrm{Aut}((B,+)/C_{(B,+)}(P_3))$ is solvable. Suppose instead that

$$P_3 \cong C_3 \times C_3.$$

Then

$$\mathrm{Aut}(P_3) \cong \mathrm{GL}_2(3).$$

Thus $(B,+)/C_{(B,+)}(P_3)$ is a subgroup of $\mathrm{GL}_2(3)$. Using GAP [7], one checks that $\mathrm{Aut}(H)$ is solvable for every subgroup $H \leq \mathrm{GL}_2(3)$. Thus $\mathrm{Aut}((B,+)/C_{(B,+)}(P_3))$ is solvable. In either case, by Lemma 2.5, there exists $k \in \mathbb{N}_{>0}$ such that

$$(B,\cdot)^{(k)} \leq C_{(B,+)}(P_3).$$

Since $(B,\cdot)$ is perfect, we conclude that

$$C_{(B,+)}(P_3) = B,$$

which implies that $P_3 \leq Z(B,+)$. Since $|P_3| = 9$, the group P_3 is abelian. We have proved that

$$P_3 \leq (B,+)' \cap Z(B,+).$$

Since $P_3 \neq 1$, this contradicts Lemma 2.6.

The case r > 7. Let R be a Sylow r-subgroup of $(B,+)$. Since $r > 7$ and $r+1$ is a power of 2, the prime r is the largest prime divisor of $|B|$, and it divides $|B|$ exactly once. Hence $|R| = r$. Since $(B,+)$ is supersolvable, its Sylow subgroup corresponding to the largest prime divisor of $|B|$ is normal. Thus R is a characteristic subgroup of $(B,+)$. We claim that

$$R \leq (B,+)'.$$

Suppose not. Then $r \mid |(B,+)/(B,+)'|$. By [8, Lemma 2.1] there is a characteristic K of $(B,+)$ such that

$$(B,+)/K \cong C_r.$$

Since K is characteristic in $(B,+)$, it is invariant under all λ_b, $b \in B$. Hence K is a left ideal of B. Therefore $(K,\cdot)$ is a subgroup of $(B,\cdot)$ of index r. We claim that

$$G := \mathrm{PSL}_2(7) \times \mathrm{PSL}_2(r)$$

has no subgroup of index r. Suppose, to the contrary, that $M \leq G$ and

$$[G : M] = r.$$

Write

$$S = \mathrm{PSL}_2(7), \qquad T = \mathrm{PSL}_2(r),$$

and identify S and T with the corresponding direct factors of G. Then

$$[S : S \cap M] = [SM : M]$$

and

$$[T : T \cap M] = [TM : M].$$

Both indices divide $[G : M] = r$, and hence each of them is either 1 or r. They cannot both be equal to 1, since otherwise $S, T \leq M$, and therefore

$$G = ST \leq M,$$

contrary to $M < G$. Thus either S or T has a subgroup of index r.

This is impossible. Indeed, since $r > 7$ and $|S| = 168$, the group S has no subgroup of index r. On the other hand, by the classification of the subgroups of $\mathrm{PSL}_2(r)$; see [10, Theorem II.8.27], the smallest index of a proper subgroup of T is $r+1$. Hence T has no subgroup of index r, proving the claim. Therefore no such K exists, and we have proved that $R \leq (B,+)'$. Now R is characteristic in $(B,+)$. Hence the lambda action induces a homomorphism

$$\lambda_R : (B,\cdot) \longrightarrow \mathrm{Aut}(R), \qquad b \mapsto \lambda_b|_R.$$

Since $R \cong C_r$, we have $\mathrm{Aut}(R) \cong C_{r-1}$, which is abelian. On the other hand, $(B,\cdot)$ is perfect. Hence every homomorphism from $(B,\cdot)$ to an abelian group is trivial. Therefore $\lambda_b(a) = a$ for every $b \in B$ and every $a \in R$. Thus

$$b \cdot a = b + a$$

for every $b \in B$ and every $a \in R$. Applying the same argument to the opposite skew brace, we also obtain

$$b \cdot a = a + b$$

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for every $b \in B$ and every $a \in R$. Hence

$$b + a = a + b$$

for all $b \in B$ and all $a \in R$. Therefore

$$R \leq Z(B, +).$$

Since $R \cong C_r$, the group R is abelian. We have already proved that $R \leq (B, +)'$, and hence

$$R \leq (B, +)' \cap Z(B, +).$$

Since $R \neq 1$, this contradicts Lemma 2.6. Therefore $(B, +)$ cannot be supersolvable. □

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