www.theoryofgroups.ir

International Journal of Group Theory
ISSN (print): 2251-7650, ISSN (on-line): 2251-7669
Vol. 3 No. 2 (2014), pp. 21-39.

www.ui.ac.ir

THE FISCHER-CLIFFORD MATRICES OF AN EXTENSION GROUP OF THE FORM $2^7{:}(2^5{:}S_6)$

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Communicated by Jamshid Moori

Abstract. The split extension group $A(4) \cong 2^7{:}Sp_6(2)$ is the affine subgroup of the symplectic group $Sp_8(2)$ of index 255. In this paper, we use the technique of the Fischer-Clifford matrices to construct the character table of the inertia group $2^7{:}(2^5{:}S_6)$ of $A(4)$ of index 63.

1. Introduction

The subgroups of symplectic groups which fix a non-zero vector of the underlying symplectic space are called *affine subgroups*. In this paper we are concerned with the inertia group $2^7{:}(2^5{:}S_6)$ in $A(4)$, the affine subgroup of $Sp_8(2)$. $A(4)$ is the maximal subgroup of $Sp_8(2)$ fixing the non-zero vector e_1 in $V_8(2)$, where $V_8(2)$ is the vector space of dimension 8 over $GF(2)$. We obtain from [1](Theorem 6.2.6 and Lemma 6.2.5) that

$$A(4) = [Sp_8(2)]_{e_1} = 2^7{:}Sp_6(2).$$

Ali [1] constructed both of 2^7 and $Sp_6(2)$ in terms of 8×8 matrices inside $Sp_8(2)$ and then act $Sp_6(2)$ on 2^7 by conjugation to represent $Sp_6(2)$ in terms of 7×7 matrices over $GF(2)$. The group $2^7{:}Sp_6(2)$ acts on $Irr(2^7)$ to produce four inertia groups $\bar{H}_i = 2^7{:}H_i$ of indices 1, 36, 28 and 63 in $2^7{:}Sp_6(2)$, respectively and where $i \in \{1,2,3,4\}$. The groups are $2^7{:}Sp_6(2)$, $2^7{:}S_8$, $2^7{:}O_6^-(2)$ and $2^7{:}(2^5{:}S_6)$, where the inertia factor groups H_i, that is, S_8, $O_6^-(2)$ and $2^5{:}S_6$ are maximal subgroups of $Sp_6(2)$. Note that $O_6^-(2) \cong U_4(2){:}2$. The reader is referred to Ali [1] regarding the above mentioned inertia groups which

MSC(2010): Primary: 20C15; Secondary: 20C40.

Keywords: Fischer-Clifford matrices, inertia group, split extension, character table, coset analysis.

Received: 20 May 2013, Accepted: 11 October 2013.

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are all split extensions and sit maximally in $2^7{:}Sp_6(2)$.

Let $\overline{G} = 2^7{:}(2^5{:}S_6) \cong \overline{H_4}$ be a split extension of $N = 2^7$ by $G = 2^5{:}S_6$, where N is the vector space of dimension 7 over $GF(2)$ on which G acts. Ali [1] has already calculated the character table of $2^7{:}Sp_6(2)$ by the method of Fischer-Clifford matrices. In this paper, the conjugacy classes and the Fischer-Clifford matrices of $2^7{:}(2^5{:}S_6)$ will be computed. We shall use the technique of the Fischer-Clifford matrices to construct the character table of $2^7{:}(2^5{:}S_6)$. Finally, the complete fusion of $2^7{:}(2^5{:}S_6)$ into $2^7{:}Sp_6(2)$ will be determined. Most of the results involving the conjugacy classes, permutation characters and character tables of the groups under consideration, were obtained by using direct computations in MAGMA [3]. Our notation may differ from the notation used in the ATLAS [5]. Motivation for the problem came from the Ph.D thesis of Ali [1].

2. **Theory of Fischer-Clifford matrices**

Let $\overline{G} = N.G$ be an extension of N by G and $\theta \in Irr(N)$. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$ for $g \in \overline{G}$ and $n \in N$ and $\theta^g \in Irr(N)$. Also define $\overline{H} = \left\{x \in \overline{G} | \theta^x = \theta\right\}$ and $H = \{g \in G | \theta^g = \theta\}$, where $\overline{H}$ is the inertia group of θ in $\overline{G}$. The inertia factor $\overline{H}/N \cong H$ can be regarded as the inertia group of θ in the factor group $\overline{G}/N \cong G$. We have that $\overline{H}$ is a subgroup of $\overline{G}$ and N is normal in $\overline{H}$. If $\overline{G}$ is a split extension then it can be easily shown that $\overline{H} = N{:}H$.

We say that θ is extendible to $\overline{H}$ if there exists $\phi \in Irr(\overline{H})$ such that $\phi\downarrow_N = \theta$. If θ is extendible to $\overline{H}$, then by Gallagher [9], we have

$$\left\{\phi | \phi \in Irr(\overline{H}), <\phi\downarrow_N, \theta> \neq 0\right\} = \left\{\beta\phi | \beta \in Irr(\overline{H}/N)\right\}.$$

Let $\overline{G}$ have the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$, $\overline{H}_i = I_{\overline{G}}(\phi_i)$, $1 \leq i \leq t$, $\phi_i \in Irr(\overline{H}_i)$ be an extension of θ_i to $\overline{H}_i$ and $\beta \in Irr(\overline{H}_i)$ such that $N \subseteq ker(\beta)$. Then it can be shown that

$$Irr(\overline{G}) = \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \mid \beta \in Irr(\overline{H}_i), N \subseteq ker(\beta)\} = \bigcup_{i=1}^{t} \{(\beta\,\phi_i)^{\overline{G}} \mid \beta \in Irr(\overline{H}_i/N)\}$$

Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H}_i$.

Let H_i be the inertia factor group and ϕ_i be an extension of θ_i to $\overline{H}_i$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H}_1 = \overline{G}$ and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. We define

$$R(g) = \{(i, y_k) \mid 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\}$$

and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Then we define the Fischer-Clifford matrix $M(g)$ by $M(g) = (a^j_{(i,y_k)})$, where

$$a^j_{(i,y_k)} = \sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\phi_i(y_{l_k}),$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by $\begin{bmatrix} C_1(g)\,M_1(g) \\ C_2(g)\,M_2(g) \\ \vdots \\ C_t(g)\,M_t(g) \end{bmatrix}$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$ in G. We can also observe that the number of irreducible characters of $\overline{G}$ is the sum of the number of irreducible characters of the inertia factors H_i's. The group $\overline{G} = 2^7{:}(2^5{:}S_6)$ is a split extension with 2^7 abelian and therefore by Mackey's theorem [9] each irreducible character of 2^7 can be extended to its inertia group in $\overline{G}$. For a thorough discussion of the theory and construction of character tables of groups of extension type, through the use of Fischer-Clifford matrices, the reader is referred to [1], [2], [11], [12], [13], [14] and [18].

3. The Conjugacy Classes of $2^7{:}(2^5{:}S_6)$

In this section we use the method of coset analysis to determine the conjugacy classes of elements of $2^7{:}(2^5{:}S_6)$. The technique of coset analysis for the conjugacy classes of the elements of group extensions of the form $\overline{G} = N.G$ where N is an abelian normal subgroup of $\overline{G}$ and $\overline{G}/N \cong G$, is applicable to both split and non-split extensions. For each conjugacy class $[g]$ in G with representative $g \in G$, we form a coset $N\overline{g}$ where $\overline{g}$ is a lifting of g in $\overline{G}$ and analyse the coset to obtain the conjugacy classes of $\overline{G}$ which correspond to the class $[g]$ of G. If $\overline{G} = N{:}G$ then the lifting of g is g itself and hence the coset $N\overline{g} = Ng$. We continue this process for all class representatives $g \in G$ and obtain all the conjugacy classes of $\overline{G}$. We refer readers to [1],[10],[13] and [18] for full details and background material regarding the method of coset analysis.

We generated $2^5{:}S_6$ by two elements $g_1 \in 4B$ and $g_2 \in 6E$ of $Sp_6(2)$. Mpono [13] constructed $Sp_6(2)$ in terms of 7×7 matrices over $GF(2)$. Here $4B$ and $6E$ are conjugacy classes of elements of $Sp_6(2)$ and g_1 and g_2 are given by :

$$g_1 = \begin{pmatrix} 1&0&0&0&0&0&0 \\ 1&0&0&0&1&1&1 \\ 0&0&1&0&1&0&0 \\ 1&1&0&1&1&1&1 \\ 1&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{pmatrix} \qquad g_2 = \begin{pmatrix} 1&0&1&1&0&0&0 \\ 0&1&1&0&1&1&1 \\ 1&0&1&0&1&0&0 \\ 0&0&0&0&1&1&1 \\ 1&0&0&1&0&0&0 \\ 1&0&1&0&0&0&1 \\ 1&0&1&0&0&1&0 \end{pmatrix}$$

The elements g_1 and g_2 are of orders 4 and 6, respectively. We computed the conjugacy classes of $2^5{:}S_6$ in MAGMA [3] and these are listed in [14]. The class representatives of each class $[g]_{2^5:S_6}$ of $2^5{:}S_6$ are given in terms of 7×7 matrices over $GF(2)$ and in total there are 37 conjugacy classes of elements.

The action of $2^5{:}S_6$ on 2^7 has eight orbits of length 1, 1, 1, 1, 30, 30, 32 and 32. The point stabilizers for the orbits of lengths 1, 30 and 32 are $2^5{:}S_6$, $2^5{:}S_4$ and S_6 of indices 1, 30 and 32, respectively in $2^5{:}S_6$. We used Programme A [1] in MAGMA [3] to compute the orbit lengths and the corresponding stabilizers. The Programme A was originally developed in CAYLEY [4] by Mpono [13] and later the programme was adapted for MAGMA by Ali [1]. We represented the group $2^5{:}S_6$ as permutations on 28 points in MAGMA, by making use of Wilson's online ATLAS of Group Representations [19]. Then the lattice of subgroups of $2^5{:}S_6$ is easily constructed using the command "SubgroupLattice(G)". We explore the structures of our point stabilizers in more detail by employing the commands "ChiefSeries(S)", "ChiefFactors(S)", "NormalSubgroups(S)" and "Complements(S, N)" in MAGMA [3] for a point stabilizer S and $N \lhd S$. The command
"IsIsomorphic(C, G_1)" confirms the isomorphism between a complement C (or any other unknown finite group) and a known finite group G_1. Hence the structures of our stabilizer groups under consideration are identified successfully.

Let $\chi(2^5 : S_6|2^7)$ be the permutation character of $2^5{:}S_6$ on 2^7. We also let $\chi(2^5{:}S_6|2^5{:}S_4)$ and $\chi(2^5{:}S_6|S_6)$ be the permutation characters of $2^5{:}S_6$ on $2^5{:}S_4$ and S_6, respectively . The permutation characters $\chi(2^5{:}S_6|2^5{:}S_4) = 1a + 5b + 9b + 15b$ and $\chi(2^5{:}S_6|S_6) = 1a + 6a + 10c + 15b$ are written in terms of the irreducible characters of $2^5{:}S_6$. The above permutation characters were computed directly in MAGMA, using the character tables of the point stabilizers together with their fusion maps into $2^5{:}S_6$.

We obtain that

$$\begin{aligned}\chi(2^5{:}S_6|2^7) &= I_{2^5:S_6}^{2^5:S_6} + I_{2^5:S_6}^{2^5:S_6} + I_{2^5:S_6}^{2^5:S_6} + I_{2^5:S_6}^{2^5:S_6} + I_{2^5:S_4}^{2^5:S_6} + I_{2^5:S_4}^{2^5:S_6} + I_{S_6}^{2^5:S_6} + I_{S_6}^{2^5:S_6} \\ &= 8 \times 1a + 2 \times 5b + 2 \times 6a + 2 \times 9a + 2 \times 10c + 4 \times 15b,\end{aligned}$$

which are the sum of the identity characters of the point stabilizers induced to $2^5{:}S_6$. We observe that the identity characters of the point stabilizers induced to $2^5{:}S_6$ are the permutation characters of $2^5{:}S_6$ on the point stabilizers.

The values of $\chi(2^5{:}S_6|2^7)$ on the different classes of $2^5{:}S_6$ determine the number k of fixed points of each $g \in 2^5{:}S_6$ in 2^7. Hence the values of k will enable us to calculate the conjugacy classes of $2^7{:}(2^5{:}S_6)$. The values of k are listed in Table 1.

TABLE 1. The values of $\chi(2^5{:}S_6|2^7)$ on the different classes of $2^5{:}S_6$

$[g]_{2^5:S_6}$	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$2G$	$2H$	$2I$	$2J$	$3A$
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_4)$	30	30	14	14	14	14	6	6	6	6	6	6
$\chi(2^5{:}S_6\|2^5{:}S_4)$	30	30	14	14	14	14	6	6	6	6	6	6
$\chi(2^5{:}S_6\|S_6)$	32	0	0	0	0	16	8	0	0	0	8	8
$\chi(2^5{:}S_6\|S_6)$	32	0	0	0	0	16	8	0	0	0	8	8
k	128	64	32	32	32	64	32	16	16	16	32	32

Table 1 (continued)

$[g]_{2^5:S_6}$	$3B$	$4A$	$4B$	$4C$	$4D$	$4E$	$4F$	$4G$	$4H$	$4I$	$4J$	$5A$
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_4)$	0	6	6	2	2	2	2	2	2	2	2	0
$\chi(2^5{:}S_6\|2^5{:}S_4)$	0	6	6	2	2	2	2	2	2	2	2	0
$\chi(2^5{:}S_6\|S_6)$	2	0	0	0	0	0	4	0	0	4	0	2
$\chi(2^5{:}S_6\|S_6)$	2	0	0	0	0	0	4	0	0	4	0	2
k	8	16	16	8	8	8	16	8	8	16	8	8

$[g]_{2^5:S_6}$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$6H$	$8A$	$8B$	$10A$	$12A$	$12B$
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_4)$	6	2	2	0	2	2	0	0	0	0	0	0	0
$\chi(2^5{:}S_6\|2^5{:}S_4)$	6	2	2	0	2	2	0	0	0	0	0	0	0
$\chi(2^5{:}S_6\|S_6)$	0	0	0	0	4	0	0	2	0	0	0	0	0
$\chi(2^5{:}S_6\|S_6)$	0	0	0	0	4	0	0	2	0	0	0	0	0
k	16	8	8	4	16	8	4	8	4	4	4	4	4

These values of k enabled us to determine the number f_j of orbits Q_i's, $1 \leq i \leq k$ which have fused together under the action of $C_{2^5:S_6}(g)$, for each class representative $g \in 2^5{:}S_6$, to form one orbit $\triangle_f$. We obtain that $2^7{:}(2^5{:}S_6)$ has exactly 219 conjugacy classes of elements after the computations of the f_j's with Programme A [1]. The authors [16] studied another group of the type $2^7{:}(2^5{:}S_6)$ in the Fischer group Fi_{22} which has 274 conjugacy classes. Hence the two groups are not isomorphic to each other. The values of the f_j's together with the lengths of each class $[\overline{g}]_{2^7:(2^5:S_6)}$ and its corresponding centralizer $C_{2^7:(2^5:S_6)}(\overline{g})$, the d_j's and the orders of each class representative $d_jg \in \overline{G}$ are listed in Table 2. The reader is referred to [1],[10],[13] and [18] for detailed information about coset analysis and the various computations involving the conjugacy classes of $2^7{:}(2^5{:}S_6)$.

TABLE 2. The conjugacy classes of elements of $G = 2^7{:}(2^5{:}S_6)$

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{2^7:(2^5:S_6)}$	$\vert[\overline{g}]_{2^7:(2^5:S_6)}\vert$	$C_{2^7:(2^5:S_6)}(\overline{g})$
$1A$	128	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$1A$	1	2949120
		$f_2=1$	$(1,1,0,1,0,0,0)$	$(1,1,0,1,0,0,0)$	$2A$	1	2949120
		$f_3=1$	$(0,0,0,0,0,1,1)$	$(0,0,0,0,0,1,1)$	$2B$	1	2949120
		$f_4=1$	$(1,1,0,1,0,1,1)$	$(1,1,0,1,0,1,1)$	$2C$	1	2949120
		$f_5=30$	$(1,1,1,1,1,1,1)$	$(1,1,1,1,1,1,1)$	$2D$	30	98304
		$f_6=30$	$(1,1,0,1,1,1,1)$	$(1,1,0,1,1,1,1)$	$2E$	30	98304
		$f_7=32$	$(1,1,1,1,1,1,0)$	$(1,1,1,1,1,1,0)$	$2F$	32	92160
		$f_8=32$	$(1,1,1,1,1,0,1)$	$(1,1,1,1,1,0,1)$	$2G$	32	92160
$2A$	64	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2H$	2	1474560
		$f_2=1$	$(0,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$2I$	2	1474560
		$f_3=16$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$4A$	32	92160
		$f_4=16$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,0,1,1)$	$4B$	32	92160
		$f_5=30$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2J$	60	49152
$2B$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2K$	60	49152
		$f_2=1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2L$	60	49152
		$f_3=3$	$(1,1,1,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$2M$	180	16384
		$f_4=3$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2N$	180	16384
		$f_5=8$	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$4C$	480	6144
		$f_6=16$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,1,1,1)$	$4D$	960	3072
$2C$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2O$	60	49152
		$f_2=1$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2P$	60	49152
		$f_3=4$	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$4E$	240	12288
		$f_4=4$	$(1,0,0,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$4F$	240	12288
		$f_5=6$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2Q$	360	8192
		$f_6=16$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,1,0,0)$	$4G$	960	3072
$2D$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2R$	120	24576
		$f_2=1$	$(0,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2S$	120	24576
		$f_3=4$	$(1,1,1,1,1,1,0)$	$(1,0,0,0,0,1,1)$	$4H$	480	6144
		$f_4=4$	$(1,1,1,1,1,0,1)$	$(1,0,0,0,0,1,1)$	$4I$	480	6144
		$f_5=6$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2T$	720	4096
		$f_6=8$	$(1,1,1,1,1,1,1)$	$(0,1,0,1,0,0,0)$	$4J$	960	3072
		$f_7=8$	$(1,1,0,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$4K$	960	3072
$2E$	64	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2U$	60	49152
		$f_2=1$	$(1,1,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$2V$	60	49152
		$f_3=1$	$(0,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2W$	60	49152
		$f_4=1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2X$	60	49152
		$f_5=8$	$(1,1,1,1,1,1,1)$	$(0,1,0,1,0,0,0)$	$4L$	480	6144
		$f_6=8$	$(1,1,1,1,1,1,0)$	$(0,1,0,1,0,0,0)$	$4M$	480	6144
		$f_7=8$	$(1,1,1,1,1,0,1)$	$(0,1,0,1,0,0,0)$	$4N$	480	6144
		$f_8=8$	$(1,1,1,1,1,0,0)$	$(0,1,0,1,0,0,0)$	$4O$	480	6144
		$f_9=12$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2Y$	720	4096
		$f_{10}=16$	$(1,1,0,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$2Z$	960	3072
$2F$	32	$f_1=1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2AA$	240	12288
		$f_2=1$	$(1,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2AB$	240	12288
		$f_3=1$‘	$(1,1,1,1,1,0,0)$	$(0,0,0,0,0,0,0)$	$2AC$	240	12288
		$f_4=1$	$(0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2AD$	240	12288
		$f_5=2$	$(1,1,1,1,0,1,0)$	$(0,0,0,0,0,0,0)$	$2AE$	480	6144
		$f_6=2$	$(1,1,1,1,0,0,1)$	$(0,0,0,0,0,0,0)$	$2AF$	480	6144
		$f_7=12$	$(1,1,1,1,1,1,1)$	$(0,1,0,1,1,0,0)$	$4P$	2880	1024
		$f_8=12$	$(1,1,1,1,1,0,1)$	$(0,1,0,1,1,0,0)$	$4Q$	2880	1024

Table 2(continued)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{2^7:(2^5:S_6)}$	$\lvert[\overline{g}]_{2^7:(2^5:S_6)}\rvert$	$C_{2^7:(2^5:S_6)}(\overline{g})$
$2G$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2AG$	480	6144
		$f_2 = 1$	$(1,1,1,1,0,1,0)$	$(1,1,0,1,0,1,1)$	$4R$	480	6144
		$f_3 = 1$	$(1,1,1,1,0,0,1)$	$(1,1,0,1,0,1,1)$	$4S$	480	6144
		$f_4 = 1$	$(0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2AH$	480	6144
		$f_5 = 6$	$(1,1,1,1,1,1,1)$	$(0,1,0,1,1,0,0)$	$4T$	2880	1024
		$f_6 = 6$	$(1,1,1,1,1,0,1)$	$(1,0,0,0,1,1,1)$	$4U$	288O	1024
$2H$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2AI$	1440	2048
		$f_2 = 1$	$(1,1,1,1,1,0,0)$	$(1,0,0,0,0,0,0)$	$4V$	1440	2048
		$f_3 = 1$	$(1,1,1,1,1,0,0)$	$(1,0,0,0,0,0,0)$	$4W$	1440	2048
		$f_4 = 1$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2AJ$	1440	2048
		$f_5 = 2$	$(1,0,0,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$4X$	2880	1024
		$f_6 = 2$	$(1,0,1,1,1,1,1)$	$(0,1,0,1,0,0,0)$	$4Y$	2880	1024
		$f_7 = 4$	$(1,1,0,1,1,0,1)$	$(1,1,0,1,1,0,0)$	$4Z$	5760	512
		$f_8 = 4$	$(1,1,1,1,1,0,1)$	$(0,1,0,1,1,0,0)$	$4AA$	5760	512
$2I$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2AK$	1440	2048
		$f_2 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$2AL$	1440	2048
		$f_3 = 1$	$(1,1,1,1,1,1,0)$	$(1,1,0,0,1,1,1)$	$4AB$	1440	2048
		$f_4 = 1$	$(1,1,1,1,1,0,1)$	$(1,1,0,0,1,1,1)$	$4AC$	1440	2048
		$f_5 = 2$	$(1,0,1,1,1,1,1)$	$(0,0,0,1,1,0,0)$	$4AD$	2880	1024
		$f_6 = 2$	$(0,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$4AE$	2880	1024
		$f_7 = 4$	$(1,1,1,1,1,1,1)$	$(0,0,1,0,0,1,1)$	$4AF$	5760	512
		$f_8 = 4$	$(1,0,1,1,1,0,1)$	$(1,1,1,1,0,0,0)$	$4AG$	5760	512
$2J$	32	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2AM$	720	4096
		$f_2 = 1$	$(1,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2AN$	720	4096
		$f_3 = 1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2AO$	720	4096
		$f_4 = 1$	$(0,0,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$2AP$	720	4096
		$f_5 = 2$	$(1,0,1,1,1,1,1)$	$(0,0,0,1,1,0,0)$	$4AH$	1440	2048
		$f_6 = 2$	$(0,1,0,1,0,1,1)$	$(0,0,0,1,1,0,0)$	$4AI$	1440	2048
		$f_7 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,0,1,1,0,0)$	$4AJ$	1440	2048
		$f_8 = 2$	$(1,1,1,1,1,0,1)$	$(0,0,0,1,1,0,0)$	$4AK$	1440	2048
		$f_9 = 4$	$(0,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$2AQ$	2880	1024
		$f_{10} = 8$	$(1,1,1,1,1,1,1)$	$(0,0,1,0,0,1,1)$	$4AL$	5760	512
		$f_{11} = 8$	$(1,1,1,1,0,0,1)$	$(0,0,1,1,1,1,1)$	$4AM$	5760	512
$3A$	32	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3A$	640	4608
		$f_2 = 1$	$(1,0,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$6A$	640	4608
		$f_3 = 1$	$(1,0,1,1,1,0,0)$	$(1,1,0,1,0,1,1)$	$6B$	640	4608
		$f_4 = 1$	$(0,1,0,1,0,1,1)$	$(0,0,0,0,0,1,1)$	$6C$	640	4608
		$f_5 = 6$	$(1,1,1,1,1,1,1)$	$(1,1,0,0,0,0,0)$	$6D$	3840	768
		$f_6 = 6$	$(1,0,1,1,0,1,1)$	$(1,1,0,1,1,0,0)$	$6E$	3840	768
		$f_7 = 8$	$(1,1,1,1,1,1,0)$	$(1,0,0,1,0,0,1)$	$6F$	5120	576
		$f_8 = 8$	$(1,1,1,1,1,0,1)$	$(1,0,0,1,0,1,0)$	$6G$	5120	576
$3B$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3B$	10240	288
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$6H$	10240	288
		$f_3 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,1,0,1,1)$	$6I$	10240	288
		$f_4 = 1$	$(0,1,1,1,1,1,1)$	$(1,1,0,1,0,1,1)$	$6J$	10240	288
		$f_5 = 2$	$(1,1,1,1,1,0,1)$	$(0,1,1,0,0,0,1)$	$6K$	20480	144
		$f_6 = 2$	$(1,1,1,1,1,1,0)$	$(0,1,1,0,0,1,0)$	$6L$	20480	144
$4A$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AN$	960	3072
		$f_2 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4AO$	960	3072
		$f_3 = 3$	$(1,1,1,1,1,1,1)$	$(0,0,0,1,0,0,0)$	$4AP$	2880	1024
		$f_4 = 3$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4AQ$	2880	1024
		$f_5 = 8$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,0,0,0)$	$8A$	7680	384

Table 2(continued)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{2^7:(2^5:S_6)}$	$\lvert[\overline{g}]_{2^7:(2^5:S_6)}\rvert$	$C_{2^7:(2^5:S_6)}(\overline{g})$
$4B$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AR$	960	3072
		$f_2 = 1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$4AS$	960	3072
		$f_3 = 3$	$(0,1,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AT$	2880	1024
		$f_4 = 3$	$(0,0,0,0,1,0,0)$	$(0,0,0,0,0,0,0)$	$4AU$	2880	1024
		$f_5 = 8$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,0,0)$	$8B$	7680	384
$4C$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AV$	2880	1024
		$f_2 = 1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$4AW$	2880	1024
		$f_3 = 2$	$(0,1,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AX$	5760	512
		$f_4 = 4$	$(1,1,0,0,0,0,1)$	$(0,0,0,0,0,0,0)$	$4AY$	11520	256
$4D$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AZ$	2880	1024
		$f_2 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BA$	2880	1024
		$f_3 = 2$	$(0,1,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BB$	5760	512
		$f_4 = 4$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4BC$	11520	256
$4E$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BD$	5760	512
		$f_2 = 1$	$(1,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$4BE$	5760	512
		$f_3 = 2$	$(1,1,1,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$4BF$	11520	256
		$f_4 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4BG$	11520	256
		$f_5 = 2$	$(0,1,1,0,0,1,0)$	$(0,0,0,0,0,0,0)$	$4BH$	11520	256
$4F$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BI$	5760	512
		$f_2 = 1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$4BJ$	5760	512
		$f_3 = 1$	$(1,1,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BK$	5760	512
		$f_4 = 1$	$(0,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$4BL$	5760	512
		$f_5 = 2$	$(1,1,0,1,0,1,0)$	$(0,0,0,0,0,0,0)$	$4BM$	11520	256
		$f_6 = 2$	$(1,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0)$	$4BN$	11520	256
		$f_7 = 4$	$(1,1,1,0,1,1,1)$	$(0,1,1,0,0,1,1)$	$8C$	23040	128
		$f_8 = 4$	$(0,0,1,0,1,0,1)$	$(0,1,1,0,0,1,1)$	$8D$	23040	128
$4G$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BO$	11520	256
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BP$	11520	256
		$f_3 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BQ$	11520	256
		$f_4 = 1$	$(0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BR$	11520	256
		$f_5 = 2$	$(0,1,1,1,1,1,0)$	$(1,1,0,1,0,0,0)$	$8E$	23040	128
		$f_6 = 2$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,0,0)$	$8F$	23040	128
$4H$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BS$	11520	256
		$f_2 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BT$	11520	256
		$f_3 = 1$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4BU$	11520	256
		$f_4 = 1$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$4BV$	11520	256
		$f_5 = 2$	$(1,1,1,1,1,1,1)$	$(0,0,0,1,1,0,0)$	$8G$	23040	128
		$f_6 = 2$	$(1,1,1,0,1,0,1)$	$(0,0,0,1,1,0,0)$	$8H$	23040	128
$4I$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4BW$	5760	512
		$f_2 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BX$	5760	512
		$f_3 = 1$	$(0,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BY$	5760	512
		$f_4 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$4BZ$	5760	512
		$f_5 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4CA$	11520	256
		$f_6 = 2$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$4CB$	11520	256
		$f_7 = 4$	$(1,1,1,1,1,1,1)$	$(0,0,0,1,1,0,0)$	$8I$	23040	128
		$f_8 = 4$	$(1,1,1,0,1,1,0)$	$(0,0,0,1,1,0,0)$	$8J$	23040	128
$4J$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4CC$	11520	256
		$f_2 = 1$	$(1,0,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$4CD$	11520	256
		$f_3 = 1$	$(1,1,0,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4CE$	11520	256
		$f_4 = 1$	$(1,1,0,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$4CF$	11520	256
		$f_5 = 2$	$(1,1,1,1,1,1,1)$	$(0,1,0,1,0,1,1)$	$8K$	23040	128
		$f_6 = 2$	$(1,1,1,1,1,1,0)$	$(0,1,0,1,0,1,1)$	$8L$	23040	128

Table 2(continued)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{2^7:(2^5:S_6)}$	$\lvert[\overline{g}]_{2^7:(2^5:S_6)}\rvert$	$C_{2^7:(2^5:S_6)}(\overline{g})$
$5A$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$5A$	36864	80
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,1,1)$	$10A$	36864	80
		$f_3 = 1$	$(0,1,1,0,1,1,1)$	$(0,0,0,0,0,1,1)$	$10B$	36864	80
		$f_4 = 1$	$(0,1,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$10C$	36864	80
		$f_5 = 2$	$(1,1,1,1,1,0,1)$	$(0,0,1,1,1,1,0)$	$10D$	73728	40
		$f_6 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,1,1,1,0,1)$	$10E$	73728	40
$6A$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6M$	1280	2304
		$f_2 = 1$	$(1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$6N$	1280	2304
		$f_3 = 4$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$12A$	5120	576
		$f_4 = 4$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,0,1,1)$	$12B$	5120	576
		$f_5 = 6$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6O$	7680	384
$6B$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6P$	7680	384
		$f_2 = 1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$6Q$	7680	384
		$f_3 = 2$	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$12C$	15360	192
		$f_4 = 4$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,1,1,1)$	$12D$	30720	96
$6C$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6R$	7680	384
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,0,0)$	$12E$	7680	384
		$f_3 = 1$	$(1,1,1,1,1,0,0)$	$(1,1,0,1,0,0,0)$	$12F$	7680	384
		$f_4 = 1$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6S$	7680	384
		$f_4 = 4$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,1,0,0)$	$12G$	30720	96
$6D$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6T$	20480	144
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6U$	20480	144
		$f_3 = 1$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$12H$	20480	144
		$f_4 = 1$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,0,1,1)$	$12I$	20480	144
$6E$	16	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6V$	7680	384
		$f_2 = 1$	$(1,1,0,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$6W$	7680	384
		$f_3 = 1$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6X$	7680	384
		$f_4 = 1$	$(1,0,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6Y$	7680	384
		$f_5 = 2$	$(1,1,1,1,1,1,1)$	$(1,0,0,0,0,0,0)$	$12J$	15360	192
		$f_6 = 2$	$(1,1,1,0,0,1,1)$	$(1,0,0,0,0,0,0)$	$12K$	15360	192
		$f_7 = 2$	$(1,1,0,1,1,1,0)$	$(1,0,0,0,0,0,0)$	$12L$	15360	192
		$f_8 = 2$	$(1,1,0,1,1,0,1)$	$(1,0,0,0,0,0,0)$	$12M$	15360	192
		$f_9 = 4$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$6Z$	30720	96
$6F$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6AA$	15360	192
		$f_2 = 1$	$(1,1,0,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6AB$	15360	192
		$f_3 = 1$	$(1,1,0,1,1,1,0)$	$(0,1,0,1,0,1,1)$	$12N$	15360	192
		$f_4 = 1$	$(1,1,0,1,1,0,1)$	$(0,1,0,1,0,1,1)$	$12O$	15360	192
		$f_5 = 2$	$(1,1,1,1,1,1,1)$	$(1,0,0,0,0,0,0)$	$12P$	30720	96
		$f_6 = 2$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$12Q$	30720	96
$6G$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6AC$	61440	48
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6AD$	61440	48
		$f_3 = 1$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$12R$	61440	48
		$f_4 = 1$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,0,1,1)$	$12S$	61440	48
$6H$	8	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6AE$	30720	96
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6AF$	30720	96
		$f_3 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$6AG$	30720	96
		$f_4 = 1$	$(1,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$6AH$	30720	96
		$f_5 = 2$	$(1,1,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$6AI$	61440	48
		$f_6 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$6AJ$	61440	48

Table 2(continued)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{2^7:(2^5:S_6)}$	$\lvert[\overline{g}]_{2^7:(2^5:S_6)}\rvert$	$C_{2^7:(2^5:S_6)}(\overline{g})$
$8A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$8K$	46080	64
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$8L$	46080	64
		$f_3 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$8M$	46080	32
$8B$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$8N$	46080	64
		$f_2 = 1$	$(0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$8O$	46080	64
		$f_3 = 2$	$(1,1,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$8P$	46080	32
$10A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$10F$	73728	40
		$f_2 = 1$	$(0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$10G$	73728	40
		$f_3 = 1$	$(1,1,1,1,1,0,1)$	$(1,1,0,1,0,1,1)$	$20A$	73728	40
		$f_4 = 1$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,1,1)$	$20B$	73728	40
$12A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$12T$	30720	96
		$f_2 = 1$	$(1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$12U$	30720	96
		$f_3 = 2$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,0,0)$	$24A$	61440	48
$12B$	4	$f_1 = 1$	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$12V$	30720	96
		$f_2 = 1$	$(1,0,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$12W$	30720	96
		$f_3 = 2$	$(1,1,1,1,1,1,0)$	$(1,1,0,1,0,0,0)$	$24B$	61440	48

4. **The Inertia groups of** $2^7{:}(2^5{:}S_6)$

The action of $2^5{:}S_6$ on the conjugacy classes of 2^7 determine 8 orbits being formed, with respective lengths of $1,1,1,1,30,30,32,32$ and 32. Therefore by Brauer's Theorem [7], $2^5{:}S_6$ acting on $Irr(2^7)$ will also form 8 orbits of lengths 1, r_1, r_2, r_3, r_4, r_5, r_6 and r_7 such that $r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7 = 127$. There are eight inertia groups $\overline{H}_i = 2^7{:}H_i$, $i = 1,2,\ldots,8$, where the H_i's are the inertia factor groups, which are maximal subgroups or sit in the maximal subgroups of $2^5{:}S_6$.

Now the sum of the number of conjugacy classes of the inertia factors must be in total equal to 219, that is, the number of the conjugacy classes of $2^7{:}(2^5{:}S_6)$. By constructing all maximal subgroups of $2^5{:}S_6$, represented as a permutation group on a set of cardinality 28, in MAGMA and taking into account the above facts, we conclude that $r_1 = 1$, $r_2 = 12$, $r_3 = r_4 = 16$ and $r_5 = 20$, $r_6 = 30$ and $r_7 = 32$. Alternatively, let the transpose of each of the generators of $G = 2^5{:}S_6$ form a matrix group T of dimension 7 over $GF(2)$. The action of T on the classes of $N = 2^7$ is the equivalent of G acting on $Irr(N)$. Hence by Brauer's Theorem we obtain 8 orbits of lengths 1, 1, 12, 16, 16, 20, 30 and 32. We obtain that $H_1 = H_2 \cong 2^5{:}S_6$, $H_3 \cong 2^4{:}S_5$, $H_4 = H_5 \cong S_6 \times 2$, $H_6 \cong (S_4 \times S_4){:}2$, $H_7 \cong 2^5{:}S_4$ and $H_8 \cong S_6$.

By represents G as a permutation group acting on 28 points within MAGMA, the command "MaximalSubgroups(G)" returns us eight maximal subgroups M_i ,i = 1,2,...8, of order 1440, 1440, 1536, 1536, 2304, 3840, 3840 and 11520, respectively. We identified the groups as $M_1 = S_6 \times 2$, $M_2 = S_6 \times 2$, $M_3 = 2^5{:}(S_4 \times 2)$, $M_4 = 2^5{:}(S_4 \times 2)$, $M_5 = 2^5{:}(3^2{:}D_4)$ $M_6 = 2^5{:}S_5$, $M_7 = 2^5{:}S_5$ and $M_8 = 2^5{:}A_6$. $M_1 \cong M_2$ is the only subgroup of G which contains $H_8 \cong S_6$ maximally. M_3 and M_4 have 53 and 40 conjugacy classes, respectively, and hence they are not isomorphic to each other. The inertia factor H_7

is the only type of subgroup of order 768 with 52 conjugacy classes in G and is a maximal subgroup of M_3. We also found that H_6 is a maximal subgroup of M_5. M_6 and M_7 are groups of $2^5{:}S_5$-type but are not isomorphic to each other, because the number of their conjugacy classes are 23 and 36, respectively. H_3 is a maximal subgroup of M_7. The same methods to identified the structures of the point stabilizers were used to determine the structures of the inertia factors and the maximal subgroups of G.

For example, the group H_7 can be constructed within M_3 in terms of permutations of a set of cardinality 28, where the following three permutations m_1, m_2, and m_3 are the generators:

m_1=(1 8 23)(2 3)(4 18 28)(6 20)(7 22)(9 12 19 10 14 17)(13 24)(15 16 26 21 27 25)

m_2=(1 28)(2 3)(4 23)(5 8)(6 22)(7 20)(11 18)(13 27)(15 26)(16 24)(17 19)(21 25)

m_3= (1 8 5 23)(2 10 14 17)(3 9 12 19)(4 18 11 28)(6 20)(7 22)(13 24)(16 26 27 25)

We construct all of its normal subgroups within MAGMA with the command "NormalSubgroups(H_7)". We identify an elementary abelian 2-group N_1 of order 32, where $N_1 = 2^5 \cong V(GF(2), 5)$ (the vector space of dimension 5 over $GF(2)$). The command "l:=Complements(H_7, N_1)" returned us 16 isomorphic copies of a group of order 24, which confirmed that H_7 is an split extension. We check that the representative subgroup "l[1]" is indeed a complement for N_1 using the commands "IsTrivial(N_1 meet l[1])". Using the command "IsIsomorphic(l[1],Sym(4))" confirmed that the complement of 2^5 in H_7 is isomorphic to the group S_4, the symmetric group of order 24 acting on a set of four points. Hence we conclude that $H_7 \cong 2^5{:}S_4$ is the structure of the inertia group H_7.

The inertia factor groups H_3, $H_4 = H_5$, H_6, H_7 and H_8 are constructed from elements within $2^5{:}S_6$ and the generators are as follows.

- $\langle \alpha_1, \alpha_2 \rangle$=$2^4{:}S_5$, $\alpha_1 \in 4F$, $\alpha_2 \in 12B$ where

$$\alpha_1 = \begin{pmatrix} 1&0&0&0&0&0&0\\ 1&0&0&0&1&1&1\\ 0&0&1&0&1&0&0\\ 1&1&0&1&1&1&1\\ 1&0&0&1&0&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0&1&1&1&1&1&1\\ 0&1&1&0&1&1&1\\ 0&0&0&1&1&0&0\\ 1&1&0&0&0&0&0\\ 1&0&0&0&1&1&1\\ 1&0&1&1&0&0&1\\ 1&0&1&1&0&1&0 \end{pmatrix}$$

- $\langle \lambda_1, \lambda_2 \rangle$= $S_6 \times 2$, $\lambda_1 \in 10A$, $\lambda_2 \in 2F$ where

$$\lambda_1 = \begin{pmatrix} 0&0&1&0&0&1&1\\ 1&0&1&1&0&0&0\\ 0&1&1&0&1&0&0\\ 0&1&0&0&0&1&1\\ 1&1&1&0&1&1&1\\ 0&0&1&1&0&1&0\\ 0&0&1&1&0&0&1 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&0&0&1&0&1&1\\ 0&1&1&1&1&0&0\\ 0&1&0&0&0&1&1\\ 0&0&0&0&1&0&0\\ 1&1&0&1&0&0&1\\ 1&1&0&1&0&1&0 \end{pmatrix}$$

- $\langle \tau_1, \tau_2, \tau_3 \rangle$= $2^5{:}S_4$, $\tau_1 \in 4F$, $\tau_2 \in 6F$, $\tau_3 \in 2D$ where

$$\tau_1=\begin{pmatrix}1&0&1&1&1&1&1\\0&0&0&0&1&0&0\\0&0&1&1&0&1&1\\0&1&1&0&0&1&1\\0&0&0&1&1&1&1\\0&0&1&0&1&1&0\\0&0&1&0&1&0&1\end{pmatrix},\ \tau_2=\begin{pmatrix}1&1&0&0&1&1&1\\0&1&0&1&0&0&0\\1&1&0&1&1&1&1\\0&1&0&0&1&1&1\\1&1&1&1&0&1&1\\0&0&1&0&1&1&0\\0&0&1&0&1&0&1\end{pmatrix}\ \tau_3=\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&1&0&1&1\\0&0&0&1&0&0&0\\0&0&0&1&1&1&1\\1&1&0&0&0&1&0\\1&1&0&0&0&0&1\end{pmatrix}$$

- $\langle\zeta_1,\zeta_2\rangle = (S_4\times S_4){:}2$, $\zeta_1\in 4E$, $\zeta_2\in 6E$ where

$$\zeta_1=\begin{pmatrix}1&0&0&1&1&0&0\\1&1&1&0&0&0&0\\1&1&1&0&1&0&0\\1&0&1&0&1&0&0\\1&0&1&1&0&0&0\\0&0&1&0&0&1&0\\0&0&1&0&0&0&1\end{pmatrix},\ \zeta_2=\begin{pmatrix}1&0&1&1&0&0&0\\0&1&0&0&0&0&0\\0&1&1&1&0&0&0\\0&0&1&0&0&0&0\\0&1&0&0&1&0&0\\0&1&1&1&0&1&0\\0&1&1&1&0&0&1\end{pmatrix}$$

- $\langle\eta_1,\eta_2\rangle = S_6$, $\eta_1\in 4I$, $\eta_2\in 6E$ where

$$\eta_1=\begin{pmatrix}1&1&0&0&0&1&1\\0&0&1&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&1&1&0&0\\1&0&1&1&1&1&1\\1&1&0&1&1&1&0\\1&1&0&1&1&0&1\end{pmatrix},\ \eta_3=\begin{pmatrix}1&0&0&0&0&0&0\\0&0&0&1&1&1&1\\0&0&1&1&1&0&0\\0&1&0&0&1&1&1\\1&0&0&1&0&0&0\\1&0&0&0&0&1&0\\1&0&0&0&0&0&1\end{pmatrix}$$

5. The Fusions of the Inertia Factor Groups into $2^5{:}S_6$

We obtain the fusions of the inertia factors H_i, $i = 3, 4, 5, 6, 7, 8$, into $2^5{:}S_6$ by using direct matrix conjugation in $2^5{:}S_6$ and the permutation characters of the inertia factor groups in $2^5{:}S_6$ of degrees 12, 16, 20, 30 and 32 respectively. MAGMA was used for the various computations. The fusion maps of the inertia factor groups into $2^5{:}S_6$ are shown in Table 3, Table 4, Table 5, Table 6 and Table 7 below.

Table 3. The fusion of $2^4{:}S_5$ into $2^5{:}S_6$

$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^4:S_5}$ $\longrightarrow$	$[g]_{2^5:S_6}$
$1A$	$1A$	$3A$	$3A$	$5A$	$5A$
$2A$	$2B$	$4A$	$4A$	$6A$	$6C$
$2B$	$2C$	$4B$	$4B$	$6B$	$6B$
$2C$	$2E$	$4C$	$4C$	$6C$	$6E$
$2D$	$2J$	$4D$	$8A$	$6B$	$8B$
$2E$	$2H$	$4E$	$4F$	$12A$	$12B$

Table 4. The fusion of $H_4 = H_5 \cong S_6 \times 2$ into $2^5{:}S_6$

$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6\times 2}$ $\longrightarrow$	$[g]_{2^5:S_6}$
$1A$	$1A$	$2F$	$2I$	$4C$	$4J$	$6D$	$6G$
$2A$	$2A$	$2G$	$2J$	$4D$	$4I$	$6E$	$6F$
$2B$	$2F$	$3A$	$3A$	$5A$	$5A$	$6F$	$6E$
$2C$	$2D$	$3B$	$3B$	$6A$	$6D$	$10A$	$10A$
$2D$	$2E$	$4A$	$4F$	$6B$	$6A$		
$2E$	$2G$	$4B$	$4H$	$6C$	$6H$		

TABLE 5. The fusion of $(S_4 \times S_4){:}2$ into $2^5{:}S_6$

$[h]_{(S_4\times S_4):2} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{(S_4\times S_4):2} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{(S_4\times S_4):2} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{(S_4\times S_4):2} \longrightarrow$	$[g]_{2^5:S_6}$
1A	1A	2E	2H	4B	4D	6A	6C
2A	2C	2F	2J	4C	4A	6B	6E
2B	2B	3A	3A	4D	4G	6C	6H
2C	2E	3B	3B	4E	4E	8A	8A
2D	2F	4A	4B	4F	4I	12A	12A

TABLE 6. The fusion of S_6 into $2^5{:}S_6$

$[h]_{S_6} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6} \longrightarrow$	$[g]_{2^5:S_6}$
1A	1A	3A	3B	5A	5A
2A	2F	3B	3A	6A	6E
2B	2E	4A	4F	6B	6H
2C	2J	4B	4I		

TABLE 7. The fusion of $2^5{:}S_4$ into $2^5{:}S_6$

$[h]_{2^5:S_4} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^5:S_4} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^5:S_4} \longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^5:S_4} \longrightarrow$	$[g]_{2^5:S_6}$
1A	1A	2M	2J	2Z	2H	4K	4H
2A	2C	2N	2H	2AA	2H	4L	4B
2B	2D	2O	2F	3A	3A	4M	4J
2C	2E	2P	2G	4A	4C	4N	4F
2D	2D	2Q	2F	4B	4D	4O	4A
2E	2B	2R	2G	4C	4E	4P	4J
2F	2A	2S	2I	4D	4E	6A	6A
2G	2E	2T	2J	4E	4F	6B	6E
2H	2H	2U	2D	4F	4H	6C	6E
2I	2C	2V	2I	4G	4I	6D	6F
2J	2B	2W	2E	4H	4G	6E	6B
2K	2I	2X	2I	4I	4I	6F	6C
2L	2J	2Y	2J	4J	4G	6G	6F

6. **The Fischer-Clifford Matrices of** $2^7{:}(2^5{:}S_6)$

Having obtained the fusion maps of the inertia factors into $2^5{:}S_6$, we are now able to compute the Fischer-Clifford matrices of the group $2^7{:}(2^5{:}S_6)$. We will use the properties in Mpono [13](Section 5.2.2)to help us in the construction of these matrices. Note that all the relations hold since 2^7 is an elementary abelian group. The following additional information obtained from Mpono [13] is sometimes needed to compute these entries:

(1) For χ a character of any group H and $h \in H$, we have $|\chi(h)| \leq \chi(1_H)$, where 1_H is the identity element of H.

(2) For χ a character of any group H and h a $p-$singular element of H, where p is a prime, then we have $\chi(h) \equiv \chi(h^p) mod p$.

(3) For any irreducible character χ of a group H and for $h_i \in C_i$ then $d_i = \frac{b_i\chi(h_i)}{\chi(1_H)}$ is an algebraic integer, where C_i is the ith conjugacy class of H and $b_i = |C_i| = [H{:}C_H(h_i)]$. It is clear if $d_i \in \mathbb{Q}$, then $d_i \in \mathbb{Z}$.

For example consider the conjugacy class $6D$ of $2^5{:}S_6$. Then we obtain that $M(6D)$ has the following form with corresponding weights attached to the rows and columns.

$$M(6D) = \begin{array}{c} \\ 36 \\ 36 \\ 36 \\ 36 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 144 & 144 & 144 & 144 \end{array} \\ \begin{pmatrix} a & e & i & m \\ b & f & j & n \\ c & g & k & o \\ d & h & l & p \end{pmatrix} \\ \begin{array}{cccc} 32 & 32 & 32 & 32 \end{array} \end{array}$$

By the results in [13](Theorem 5.2.4 and property (e) of the Fischer-Clifford matrices), we have $a = e = i = m = 1$, $b = c = d = 1$. Thus we get the following form

$$M(6D) = \begin{array}{c} \\ 36 \\ 36 \\ 36 \\ 36 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 144 & 144 & 144 & 144 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & f & j & n \\ 1 & g & k & o \\ 1 & h & l & p \end{pmatrix} \\ \begin{array}{cccc} 32 & 32 & 32 & 32 \end{array} \end{array}$$

By the orthogonality relations for columns and rows in [13](Proposition 5.2.3 and Proposition 5.2.7) and the Fischer-Clifford properties we obtained the equations $f + g + h = -1$, $f^2 + g^2 + h^2 = 3$, $j + k + l = -1$, $j^2 + k^2 + l^2 = 3$, $n + o + p = -1$, $n^2 + o^2 + p^2 = 3$, $f + j + n = -1$, $f^2 + j^2 + n^2 = 3$, $g + k + o = -1$, $g^2 + k^2 + o^2 = 3$, $h + l + p = -1$ and $h^2 + l^2 + p^2 = 3$. Solving the above equations simultaneous and also taking into consideration the additional information discussed above, we obtained the following six possibilities for the remaining entries in $M(6D)$:

(1) $\{f = 1, g = -1, h = -1, j = -1, k = 1, l = -1, j = -1, o = -1, p = 1\}$
(2) $\{f = 1, g = -1, h = -1, j = -1, k = -1, l = 1, j = -1, o = 1, p = -1\}$
(3) $\{f = -1, g = 1, h = -1, j = -1, k = -1, l = 1, j = 1, o = -1, p = -1\}$
(4) $\{f = -1, g = 1, h = -1, j = 1, k = -1, l = -1, j = -1, o = -1, p = 1\}$
(5) $\{f = -1, g = -1, h = 1, j = 1, k = -1, l = -1, j = -1, o = 1, p = -1\}$
(6) $\{f = -1, g = -1, h = 1, j = -1, k = 1, l = -1, j = 1, o = -1, p = -1\}$

Since in $2^7{:}(2^5{:}S_6)$ we have $(6U)^3 = 2I$, $(12H)^3 = 4B$ and $(12I)^3 = 4A$, for $\chi \in Irr(2^7{:}(2^5{:}S_6))$ we must have $\chi(2I) \equiv \chi(6U)(mod 3)$, $\chi(4B) \equiv \chi(12H)(mod 3)$ and $\chi(4A) \equiv \chi(12I)(mod 3)$. Checking the validity of these congruent relations for the portions of the character table of $2^7{:}(2^5{:}S_6)$ corresponding to $M(2A)$ and to the six candidates of $M(6D)$ we deduce that

$$M(6D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix} \text{ is the only candidate.}$$

For each class representative $g \in 2^5{:}S_6$, we construct a Fischer-Clifford matrix $M(g)$ which are listed in Table 8.

Table 8. The Fischer-Clifford Matrices of $2^7{:}(2^5{:}S_6)$

$M(g)$	$M(g$
$M(1A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\ 12 & 12 & -12 & -12 & 4 & -4 & 0 & 0 \\ 16 & -16 & 16 & -16 & 0 & 0 & -4 & 4 \\ 16 & -16 & 16 & -16 & 0 & 0 & 4 & -4 \\ 20 & 20 & -20 & -20 & -4 & 4 & 0 & 0 \\ 30 & 30 & 30 & 30 & -2 & -2 & 0 & 0 \\ 32 & -32 & -32 & 32 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(2A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 16 & -16 & 4 & -4 & 0 \\ 16 & -16 & -4 & 4 & 0 \\ 30 & 30 & 0 & 0 & -2 \end{pmatrix}$
$M(2B)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ 4 & -4 & -4 & 4 & 0 & 0 \\ 12 & -12 & 4 & -4 & 0 & 0 \\ 2 & 2 & 2 & 2 & -2 & 0 \\ 12 & 12 & -4 & -4 & 0 & 0 \end{pmatrix}$	$M(2C)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ 8 & -8 & -4 & 4 & 0 & 0 \\ 8 & -8 & 4 & -4 & 0 & 0 \\ 2 & 2 & -2 & -2 & 2 & 0 \\ 12 & 12 & 0 & 0 & -4 & 0 \end{pmatrix}$
$M(2E)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 8 & -8 & 8 & -8 & 4 & -4 & 0 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & -4 & 4 & 0 & 0 \\ 8 & -8 & 8 & -8 & -4 & 4 & 0 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 4 & -4 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 12 & 12 & 12 & 12 & 0 & 0 & 0 & 0 & -4 & 0 \\ 16 & 16 & -16 & -16 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(2D)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 8 & -8 & -4 & 4 & 0 & 0 & 0 \\ 8 & -8 & 4 & -4 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 12 & 12 & 0 & 0 & -4 & 0 & 0 \end{pmatrix}$
$M(2F)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 4 & -4 & 4 & -4 & 4 & -4 & 0 & 0 \\ 4 & -4 & 4 & -4 & -4 & 4 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & -3 & -3 & -1 & 1 \\ 3 & 3 & 3 & 3 & 3 & 3 & -1 & -1 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(2G)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 4 & -4 & 4 & -4 & 0 & 0 \\ 4 & 4 & -4 & -4 & 0 & 0 \\ 3 & 3 & 3 & 3 & -1 & -1 \\ 3 & -3 & -3 & 3 & -1 & 1 \end{pmatrix}$
$M(2H)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 4 & 4 & -4 & -4 & 0 & 0 & 0 & 0 \\ 4 & -4 & 4 & -4 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 2 & 2 & 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & -2 & 2 & 0 & 0 \end{pmatrix}$	$M(2I)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\ 4 & -4 & -4 & 4 & 0 & 0 & 0 & 0 \\ 4 & -4 & 4 & -4 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & 2 & -2 & -2 & -2 & 2 & 0 & 0 \\ 2 & 2 & 2 & 2 & -2 & -2 & 0 & 0 \end{pmatrix}$
$M(2J)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 4 & -4 & 4 & -4 & 4 & -4 & 0 & 0 & 0 & 0 & 0 \\ 4 & -4 & -4 & 4 & 0 & 0 & 4 & -4 & 0 & 0 & 0 \\ 4 & -4 & 4 & -4 & -4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & -4 & -4 & 4 & 0 & 0 & -4 & 4 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 2 & 2 & 2 & 2 & -2 & -2 & 2 & 2 & -2 & 0 & 0 \\ 2 & 2 & 2 & 2 & -2 & -2 & -2 & -2 & 2 & 0 & 0 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(3A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 6 & 6 & -6 & -6 & 2 & -2 & 0 & 0 \\ 4 & -4 & 4 & -4 & 0 & 0 & -2 & 2 \\ 4 & -4 & 4 & -4 & 0 & 0 & 2 & -2 \\ 2 & 2 & -2 & -2 & -2 & 2 & 0 & 0 \\ 6 & 6 & 6 & 6 & -2 & -2 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 & 0 \end{pmatrix}$

Table 8 (continued)

$M(g)$	$M(g)$
$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & -2 & 2 & -2 & 0 & 0 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 \\ 2 & -2 & -2 & 2 & 0 \\ 6 & -6 & 2 & -2 & 0 \\ 6 & 6 & -2 & -2 & 0 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & -6 & 2 & -2 & 0 \\ 2 & -2 & -2 & 2 & 0 \\ 6 & 6 & -2 & -2 & 0 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 4 & -4 & 0 & 0 \\ 2 & 2 & -2 & 0 \end{pmatrix}$
$M(4D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 4 & -4 & 0 & 0 \\ 2 & 2 & -2 & 0 \end{pmatrix}$	$M(4E) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 4 & -4 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 \end{pmatrix}$
$M(4F) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 4 & -4 & -4 & 4 & 0 & 0 & 0 & 0 \\ 2 & -2 & 2 & -2 & 2 & -2 & 0 & 0 \\ 2 & -2 & 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 4 & 4 & -4 & -4 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(4G) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 \end{pmatrix}$
$M(4H) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 \end{pmatrix}$	$M(4I) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & -2 & 2 & -2 & 2 & -2 & 0 & 0 \\ 4 & -4 & -4 & 4 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 4 & 4 & -4 & -4 & 0 & 0 & 0 & 0 \end{pmatrix}$
$M(4J) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & 2 & -2 & 0 & 0 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 2 & -2 & 0 \\ 4 & -4 & -2 & 2 & 0 \\ 6 & 6 & 0 & 0 & -2 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 4 & -4 & 0 & 0 \\ 2 & 2 & -2 & 0 \end{pmatrix}$
$M(6C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & -2 & 0 \\ 2 & -2 & 2 & -2 & 0 \\ 2 & -2 & -2 & 2 & 0 \end{pmatrix}$	$M(6D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$

Table 8 (continued)

$M(g)$	$M(g)$
$M(6E)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 2 & 2 & -2 & -2 & 0 & -2 & 0 & 2 & 0 \\ 2 & -2 & 2 & -2 & -2 & 0 & 2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 0 & 2 & 0 & -2 & 0 \\ 2 & -2 & 2 & -2 & 2 & 0 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 4 & -4 & -4 & 4 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(6F)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \end{pmatrix}$
$M(6G)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(6H)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 0 & 0 \end{pmatrix}$
$M(8A)=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(8B)=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(10A)=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(12A)=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(12B)=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	

7. Character Table of $2^7{:}(2^5{:}S_6)$

The Fischer-Clifford matrix $M(g)$ will be partioned row-wise into blocks, where each block corresponds to an inertia group $\overline{H}_i$. Using the columns of the character table of the inertia factor H_i of $\overline{H}_i$ which correspond to the classes of H_i fusing to the class $[g]$ in G and multiplying these columns by the rows of the Fischer-Clifford matrix $M(g)$ which correspond to $\overline{H}_i$, we fill the portion of the character table of $\overline{G}$ which is in the block corresponding to $\overline{H}_i$ for the classes of $\overline{G}$ coming from the coset Ng.

The character table of $\overline{G}$ will be partitioned row-wise into blocks B_i, where each block corresponds to an inertia group $\overline{H}_i = 2^7{:}H_i$. Therefore $Irr(\overline{G} = 2^7{:}(2^5{:}S_6)) = \bigcup_{i=1}^{8} B_i$, where $B_1 = \{\chi_j | 1 \leq j \leq 37\}$, $B_2 = \{\chi_j | 38 \leq j \leq 74\}$, $B_3 = \{\chi_j | 75 \leq j \leq 92\}$, $B_4 = \{\chi_j | 93 \leq j \leq 114\}$, $B_5 = \{\chi_j | 115 \leq j \leq 136\}$, $B_6 = \{\chi_j | 137 \leq j \leq 156\}$, $B_7 = \{\chi_j | 157 \leq j \leq 208\}$ and $B_8 = \{\chi_j | 209 \leq j \leq 219\}$. The precise description of this method is described in [1], [2], [11], [12], [13] and [18]. The character table of $2^7{:}(2^5{:}S_6)$ can be found in [14] or the reader can obtain it directly from the authors. The consistency and accuracy of the character table of $2^7{:}(2^5{:}S_6)$ have been tested by using Programme C [14] written in GAP [17].

8. The Fusion of $2^7{:}(2^5{:}S_6)$ into $2^7{:}Sp_6(2)$

We use the results of Section 3 to compute the power maps of the elements of $2^7{:}(2^5{:}S_6)$ which are found in [14].

We are able to obtain the partial fusion of $2^7{:}(2^5{:}S_6)$ into $2^7{:}Sp_6(2)$ by using the information provided by the conjugacy classes of the elements of $2^7 : (2^5{:}S_6)$ and $2^7{:}Sp_6(2)$, and the fusion map of $2^5{:}S_6$ into $Sp_6(2)$. We used the technique of set intersections for characters to restrict the irreducible characters $63a$, $63b$, $36a$, $36b$, $28a$ and $28b$ of $2^7{:}Sp_6(2)$ to $2^7{:}(2^5 : S_6)$ to determine fully the fusion of the classes of $2^7{:}(2^5{:}S_6)$ into $2^7{:}Sp_6(2)$. Hence based on the partial fusion of $2^7{:}(2^5{:}S_6)$ into $2^7{:}Sp_6(2)$ which has already been determined , we obtain that

$$
\begin{aligned}
(63a)_{2^7:(2^5:S_6)} &= \chi_{38} + \chi_{157} + \chi_{209} \\
(63b)_{2^7:(2^5:S_6)} &= \chi_{39} + \chi_{160} + \chi_{210} \\
(36a)_{2^7:(2^5:S_6)} &= \chi_{115} + \chi_{137} \\
(36b)_{2^7:(2^5:S_6)} &= \chi_{118} + \chi_{139} \\
(28a)_{2^7:(2^5:S_6)} &= \chi_{75} + \chi_{93} \\
(28b)_{2^7:(2^5:S_6)} &= \chi_{76} + \chi_{94}
\end{aligned}
$$

We refer the reader for detailed information regarding the above set intersections technique to Ali [1], Ali and Moori [2], Moori [11], Moori and Mpono [12], Mpono [13] and Prins [15].

Using the partial fusion already determined and the values of $63a$, $63b$, $36a$, $36b$, $28a$ and $28b$ on the classes of $2^7{:}Sp_6(2)$ and the values of $(63a)_{2^7:(2^5:S_6)}$, $(63a)_{2^7:(2^5:S_6)}$, $(36a)_{2^7:(2^5:S_6)}$, $(36b)_{2^7:(2^5:S_6)}$, $(28a)_{2^7:(2^5:S_6)}$ and $(28b)_{2^7:(2^5:S_6)}$ on the classes of $2^7{:}(2^5{:}S_6)$, we are able to complete the fusion map of $2^7{:}(2^5{:}S_6)$ into $2^7{:}Sp_6(2)$ and is given in [14].

Acknowledgments

The first author would like to thank his supervisor (second author) for his advice and support. The financial support from the NRF and University of the Western Cape are also acknowledged. I am most grateful to my Lord Jesus Christ.

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